Peace through bribing*

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Abstract

We study a model in which before a conflict between two parties escalates into a war (in the form of an all-pay auction), a party can offer a take-it-or-leave-it bribe to the other for a peaceful settlement. In contrast to the received literature, we find that peace security is impossible in our model. We characterize the necessary and sufficient conditions for peace implementability. Furthermore, we find that separating equilibria do not exist and the number of (on-path) bribes in any non-peaceful equilibria is at most two. We also consider a requesting model and characterize the necessary and sufficient conditions for the existence of robust peaceful equilibria, all of which are sustained by the identical (on-path) request. Contrary to the bribing model, peace security is possible in the requesting model.

Keywords: All-pay auction; Bribing; Signaling; Endogenous conflict.

JEL codes: D44; D74; D82.

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1 Introduction

It is not uncommon that before a conflict between two parties escalates into a war, one of them may seek a peaceful settlement by bribing the opponent.[1] Under what conditions would the briber always succeed and thus peace be guaranteed? To shed light on the question, we consider a model in which before a single-object all-pay auction starts, a player (player 1, he) can offer a take-it-or-leave-it bribe to the other player (player 2, she). If player 2 accepts the bribe, then she exits the auction and player 1 wins the object at zero price and thus peace is achieved; otherwise, both players enter the auction and compete with each other non-cooperatively and thus the conflict escalates into a war.

In our model, there can be various degrees of peace prospects, and we adopt similar notions of peace prospects from Zheng (2019b). If peace occurs with certainty in a perfect Bayesian equilibrium (hereafter, equilibrium), then we say that the equilibrium is *peaceful*. If there exists a peaceful equilibrium (supported by a belief system), then we say that peace is *implementable*. Furthermore, if an equilibrium survives the D1 criterion (Cho and Sobel (1990)), it is said to be robust. Finally, if peace occurs with certainty for any belief system, then peace is securable.

In contrast to Zheng (2019b), we find that in the bribing model, peace security is impossible. The reasons for this qualitatively different result are the following. In Zheng (2019b), if a player rejects the proposal by the mediator, the player does not have an opportunity to offer a new proposal and both players enter the continuation auction immediately. So security requires that the highest possible expected payoff of each player from the continuation auction (among all possible beliefs) is lower than the proposed share of the prize, should the player reject the proposal. The highest expected payoff of the rejecting player is achieved when he/she is believed to be of the lowest type. So basically, in Zheng (2019b), the proposal of the mediator is meant to remove the strong type's incentive to pretend to be weak and bid low and win the object at a low cost in the continuation auction. In our model, the strategic consideration of player 1 is very different when deviating to off-path actions. Player 1 has the opportunity and incentive to propose a bribe slightly lower than the on-path bribe, in a hope that the new bribe is still high enough and only leads to a minor probability of rejection. This hope may be fulfilled when player 2 believes the new bribe is offered by high enough types of player 1 (e.g., the highest type). It is then possible that cost-saving effect from the lower bribe (when accepted) dominates the competition effect from the higher chance of rejection and the resultant war. Peace security requires that this should not be possible for any belief system. However, it turns out that player

[1] Consider legal disputes, and international conflicts for examples.

1 can always deviate to a slightly lower bribe and for some high enough belief about player 1's type, the bribe would be accepted by player 2 with certainty. That means a profitable deviation for player 1 and thus the impossibility of peace security.

Peace implementability is however possible in our model because it only requires some specific belief system to sustain a peaceful equilibrium. We characterize the necessary and sufficient conditions for peace implementability. With the conditions, generally, there exists a continuum of equilibria, which are all robust and yield different payoffs for the players.

We then consider comparative statics for peace implementability. In Zheng (2019b), if either of the two players becomes stronger in the sense that the type distribution becomes more first-order stochastic dominant and the support is unchanged, then peace implementability is preserved. In our model, this result remains true for player 1's type distribution but not necessarily true for player 2's. The reason is that first-order stochastic dominance means a redistribution of probability mass from lower to higher types. Thus it is possible that the lowest type of player 1 can earn a higher expected payoff by offering a higher bribe. The higher bribe may lead to a significant increase in the probability of acceptance, the effect of which may outweigh the impact of a higher payment (when the bribe is accepted). That means a higher maximum off-path expected payoff of player 1, which renders peace implementability impossible.

We proceed to examine non-peaceful equilibria. We find that in any non-peaceful equilibrium either there is a pooling bribe or there are two bribes. In the latter case, the lower bribe is rejected with a positive probability while the higher bribe is accepted with certainty. In particular, we show that there exist no separating equilibria. This result echoes with that of Rachmilevitch (2013),[2] but it is in contrast to the finding of Esö and Schummer (2004). The intuition is that a higher type always has the incentive to mimic a marginally lower type. When doing so by offering a lower bribe, a higher type can benefit not only from acceptance but also from the rejection of the lower bribe because in the continuation auction the highest bid is lower and the higher type wins for sure. These two effects outweigh the small cost of having to compete with slightly more types in the continuation auction.

It is also common in practice that one party requests a payment from the other in exchange for its own exit from the conflict. We further discuss briefly a requesting model in which player 1 can commit to a bargaining protocol in which he only requests a bribe from his opponent. We characterize the necessary and sufficient conditions for the existence of robust peaceful equilibria. Contrary to the bribing model, all robust peaceful equilibria share the same on-path request–the lowest valuation of the opponent (which is the highest possible peaceful request).

[2] Rachmilevitch (2013) analyzes the first-price auction case and his attention is not on peaceful settlement.

The intuition for this result is that upon receiving any off-path request, under the D1 criterion, the only reasonable belief of player 2 is that it is sent from the highest type of player 1. So any request lower than the lowest valuation of player 2 is accepted by all types of player 2. So if the equilibrium request is lower than the lowest valuation of player 2, then there is a gap between the equilibrium request and the lowest valuation of player 2. And then player 1 can always deviate to a request in the middle, which would be accepted with probability one. Hence, the only possible equilibrium request should be the lowest valuation of player 2.

Interestingly, however, we find that in the requesting model peace security is possible and we characterize the necessary and sufficient conditions for it. Peace security does not require more restrictive conditions on player 1's primitive than the ones for a robust peaceful equilibrium if the lowest type of player 1 is higher than the counterpart of player 2, while it does require some more restrictive conditions on player 1's primitive if the opposite is true. The reason is that in the latter case, without the extra condition it could be possible for the highest type of player 1 to trick player 2 with the lowest belief into rejecting the request and submitting a low bid in the continuation auction. But in the former case, the lowest type of player 1 is high enough so that the possibility vanishes.

The rest of the article is organized as follows. A brief review of related literature is given for the rest of this section. In section 2, we describe the model and introduce some results for all-pay auctions from Zheng (2019b). In section 3, we first show that peace security is impossible and then characterize the necessary and sufficient conditions for peace implementability. We then proceed to consider non-peaceful equilibria. In section 4, we discuss the requesting model briefly and characterize the necessary and sufficient conditions for the existence of robust peaceful equilibria. Section 5 concludes.

1.1 Related literature

Our work is most related to Esö and Schummer (2004) and Zheng (2019b). Esö and Schummer (2004) pioneered the literature of dynamic models for bidder collusion. They consider second-price auctions and are interested in the notion of bribe-proof, namely the possibility that peaceful settlement cannot be achieved (in some equilibria). They show that under some regularity conditions there exist some robust increasing equilibria. The difference is that we consider all-pay auctions and we are interested in the possibilities of a peaceful settlement. The nature of all-pay auctions complicates the analysis because there are no (weakly) dominant strategies available for players in the continuation auctions should player 2 reject a bribe. On the other hand, thanks to Zheng (2019b), the analysis in the current article becomes possible with his results on two-player all-pay auctions (with arbitrary, independent type distributions).

Since Esö and Schummer (2004)'s work, alternative models for second-price and first-price auctions have been proposed. Rachmilevitch (2013) considers the same bargaining protocol in first-price auctions while Rachmilevitch (2015) allows for alternating offers between two players in second-price auctions. Troyan (2017) extends Esö and Schummer (2004)'s model to a setting of interdependent valuations and affiliated signals. Our article Lu et al. (2021) considers again second-price auctions but allows for a combination of a bribe and a request. The difference of the current paper from this strand of literature is that we consider all-pay auctions and our focus is on peaceful settlement.

Zheng (2019b) considers a conflict-mediation model in which before an all-pay auction starts, a mediator can recommend a split of the prize. If the split is accepted by both players, then the game ends and peace is achieved; otherwise, both players enter the auction and compete with each other non-cooperatively. The difference is that in our model a player is endowed with bargaining power and bargains with the opponent directly.[3] A consequence of the difference is that in our model the briber can propose different off-path bribes which endogenize player 2's beliefs and replies. In particular, in the continuation games of our model, both players' type distributions may be updated whereas in his model only the rejector's type distribution is updated. Zheng (2019a) considers the same mediation model for first-price auctions. Kamranzadeh and Zheng (2022) revisit the all-pay auction model and explore the case where peace cannot be guaranteed. In their paper, the mediator's objective is to maximize the sum of the ex-ante payoffs of the players.

More generally, our work is related to the conflict mediation literature. In one strand of the literature, such as Bester and Wärneryd (2006), Compte and Jehiel (2009), Fey and Ramsay (2011), Hörner et al. (2015), and Spier (1994), an exogenous outside option of peace is assumed for the conflicting parties and there is a mediator who chooses a mechanism to preempt the conflict. In another strand of the literature, the outside option is rather determined endogenously by the continuation play during the conflict if the mediation does not preempt the conflict. For example, apart from Zheng (2019b), Balzer and Schneider (2021) consider a particular mechanism–the Alternative Dispute Resolution (ADR). Assuming discrete type distributions, they characterize the optimal ADR mechanism that maximizes the settlement rate.

Some works study conflict-avoidance in complete-information settings and typically model wars as Tullock contests. For example, Beviá and Corchón (2010) consider a stylized Tullock

[3]Another difference from Zheng (2019) is that we assume that players have private valuations whereas in his model the valuations are common but bidding costs are private.

contest before which each of the two parties may pay the opponent for peace. They study the roles of inequality of resources and military proficiency in the bargaining outcomes of peaceful agreements. With a potential war also modeled as a Tullock contest, Herbst et al. (2017) examine two games–a split game with a mediator and a demand game without a mediator. Their laboratory results suggest that in the split game the likelihood of conflict is independent of the balance of power whereas in the demand game the likelihood is positively correlated with imbalance of power. In Kimbrough and Sheremeta (2013), a player can make an offer to the opponent before the contest starts. The offer can be binding or non-binding (the recipient can still engage the opponent even after receiving the payment). They find in their experiments that even in the non-binding case the ability to offer side-payments still reduces the cost of conflict.

Finally, Hörner and Sahuguet (2007) analyses a dynamic auction in which jump bidding is used by players to signal their private information. They show that non-monotonic signaling is possible when jump bidding is allowed.

2 The model

Two risk neutral players, player 1 (he) and 2 (she), are about to attend an all-pay auction before which player 1 can offer a take-it-or-leave-it bribe b to player 2. If player 2 accepts the bribe, then she exits the auction and player 1 wins the object with zero price; otherwise, both players enter into the auction and compete with each other non-cooperatively.

For $i \in \{1,2\}$, player i's valuation v_i (referred as *type* as well below) for the auctioning object is independently distributed according to an atomless continuous F_i with density function f_i on a support (denoted by supp below) $[\underline{v}_i, \bar{v}_i]$ with $\bar{v}_i > \underline{v}_i$.

We focus on perfect Bayesian equilibria in which player 1 uses a pure (behavioral) strategy for the bribes and player 2 also uses a pure strategy when deciding whether to accept or reject. However, we allow for mixed strategy equilibria for the continuation auction in the same sense as in Zheng (2019b).

Upon receiving an off-path bribe b, player 2 forms a belief $\tilde{F}_1$ about player 1's type and uses a behavioral acceptance-rejection strategy ϱ–some types accept it and the other types reject it. The induced type distribution of player 2 in the continuation auction is denoted by $\tilde{F}_2$. We assume that $\tilde{F}_1$ and $\tilde{F}_2$ are independent. Each type v_i is allowed to use a mixed strategy and thus player i uses a *distributional strategy* which is a probability measure on the product space of types and bids. We use σ to denote the equilibrium (and the strategy pair) of a continuation auction $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$. To distinguish σ from the equilibrium of the grand game, we use BNE

(Bayesian Nash equilibrium) for σ. For convenience of exposition, we use π_i to denote the payoffs from the grand game and U_i to denote the payoffs from the continuation auctions. The support of $\tilde{F}_i$ is denoted by V_i. Notation δ_x is the Dirac measure (and the distribution) with support $\{x\}$.

Player i's strategy in a BNE σ yields a bid distribution $H_{i,\sigma}(\cdot)$ and $c_{i,\sigma} := H_{i,\sigma}(0)$. So if player $-i$ bids β, the win probability is $H_{i,\sigma}(\beta)$. Due to the payment rule of an all-pay auction and the equilibrium condition, for any σ, the supports of the bid distributions of the two players are common and we denote the highest possible bid by x_σ.

For off-path consistency, it is required that ϱ and $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$ are consistent with each other:

(i). The acceptance-rejection behavior of player 2's types in ϱ is consistent with the equilibrium outcome in $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$ (and belief $\tilde{F}_1$); if type $v_2 \notin V_2$ accepts an off-path bribe b, she should not find it profitable to submit any bid in the continuation auction; if type $v_2 \in V_2$ rejects an off-path bribe b, then her payoff from the continuation auction should not be lower than b.

(ii). $\tilde{F}_2$ is consistent with ϱ and the equilibrium outcome from $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$, and is common for both players.[4]

Off-path consistency does not rule out the following possibilities. For an off-path bribe b and a belief $\tilde{F}_1$, there may exist multiple pairs of ϱ and σ that satisfy the requirements, which we call consistent replies. If two different consistent replies yield the same expected payoff for each type v_i $(i = 1, 2)$ in the grand game, we say they are *equivalent*. We discuss equivalent replies with more details in section 3.

Off-path consistency implies that if for an off-path bribe the lowest rejecting type is in the interior of the type support, then this type must be indifferent between accepting the bribe and rejecting it. The indifference condition in turn implies a unique consistent reply for each belief induced by strictly positive off-path bribes.

Finally, for the continuation auction $\mathcal{G}(F_1, \tilde{F}_2)$ following rejection of an on-path pooling bribe, we use σ_2 to denote an associated BNE.

2.1 Preliminaries

In this section we introduce some properties of equilibria of two-player all-pay auctions (with independent type distributions) from Zheng (2019b).

[4]Given the rejection set implied by ϱ, Bayes's rule is applicable.

Lemma 1. *For any all-pay auction $\mathcal{G}(\tilde{F}_1,\tilde{F}_2)$ and any BNE σ of $\mathcal{G}(\tilde{F}_1,\tilde{F}_2)$ there exists a unique triple $(x_\sigma, c_{1,\sigma}, c_{2,\sigma}) \in \mathbb{R}_{++} \times [0,1]^2$ such that for each $i \in \{1,2\}$ and all $b \in [0, x_\sigma]$, $H_{i,\sigma}(x_\sigma) = 1$ and*

$$H_{i,\sigma}(\beta) = c_{i,\sigma} + \int_0^\beta \frac{1}{\tilde{F}_{-i}^{-1}(H_{-i,\sigma}(y))} dy,$$

where $c_{i,\sigma} := H_{i,\sigma}(0)$ and

$$\tilde{F}_i^{-1}(x) := \inf\{t \in \text{supp } \tilde{F}_i : \tilde{F}_i(t) \geq x\}.$$

Furthermore,

(a). $c_{1,\sigma} c_{2,\sigma} = 0$*;*

(b). given the equilibrium behavior of types $v_{-i} \in \text{supp } \tilde{F}_{-i}$ in σ, for any type v_i, the supremum expected payoff, denoted by $U_i(v_i|\sigma)$, is[5] [6]

$$U_i(v_i|\sigma) = \max_{\beta \in R_+} H_{-i,\sigma}(\beta) v_i - \beta.$$

Proof. Theorem 6 and Theorem 5 in Zheng (2019b). □

Remark 1. A useful interpretation of $c_{1,\sigma}c_{2,\sigma} = 0$ is that the lowest types of $\tilde{F}_1, \tilde{F}_2$ (denoted by say, $\underline{v}_1, \underline{v}_2$) should not both earn positive expected payoffs because $U_i(\underline{v}_i|\sigma) = c_{-i,\sigma}\underline{v}_i$. In particular, the highest bid x_σ should not be lower than $\min\{\underline{v}_1, \underline{v}_2\}$.

Let

$$\mathcal{E}_i(\tilde{F}_i) := \text{ the set of BNEs of } \mathcal{G}(\tilde{F}_i, \tilde{F}_{-i}).$$

The following lemma follows from Lemma 1 and concerns the equilibria of one-sided complete information all-pay auctions.

[5] The result is for any v_i, including $v_i \notin \text{supp } \tilde{F}_i$ (if they can bid secretly). In Zheng (2019b), for this result, type v_i is restricted to strictly positive values, which is due to the fact that in his common value model, type zero's valuation is one, although the cost of any positive bid is infinite. So when $c_{-i} > 0$, type zero bids zero but earns an expected payoff $c_{-i}/2$ due to the tie-breaking rule whereas any positive type can earn an expected payoff c_{-i} by bidding 0^+. Therefore, type zero should be excluded for this result. In our model, the valuations are different for different types. So when $c_{-i} > 0$, the expected payoff of any type $v_i > 0$ from bidding 0^+ is $c_{-i}v_i$, which converges to zero when v_i tends to zero. Thus, in our model, for this result, type zero need not to be excluded.

[6] As is common in the literature (e.g., Lebrun (1996) and Zheng (2019b)), if player $-i$'s bid distribution has an atom at zero bid, then the tie-breaking rule is assumed in such a way that when player i bids zero the win probability of player i is also $H_{-i,\sigma}(0)$. That is because the supremum expected payoff of player i is $H_{-i,\sigma}(0)v_i$ by bidding 0^+.

Lemma 2. *For any $i \in \{1,2\}$, and any $v_i^* \neq 0$, $\mathcal{E}_i(\delta_{v_i^*}) = \{\sigma^*\}$ such that*

$$\forall \beta \in [0, x_{\sigma^*}] : H_{i,\sigma^*}(\beta) = c_{i,\sigma^*} + \int_0^\beta \frac{1}{\tilde{F}_{-i}^{-1}\left(\frac{s}{v_i^*} + c_{-i,\sigma^*}\right)} ds; \tag{1}$$

$$\forall \beta \in [0, x_{\sigma^*}] : H_{-i,\sigma^*}(\beta) = \frac{\beta}{v_i^*} + c_{-i,\sigma^*};$$

$$c_{i,\sigma^*} c_{-i,\sigma^*} = 0;$$

$$x_{\sigma^*} = v_i^*(1 - c_{-i,\sigma^*}).$$

Proof. Lemma 13 in Zheng (2019b). □

Remark 2. The boundary condition $H_{i,\sigma^*}(x_{\sigma^*}) = 1$ implies

$$c_{i,\sigma^*} = 1 - v_i^* \int_{c_{-i,\sigma^*}}^1 \frac{1}{\tilde{F}_{-i}^{-1}(s)} ds. \tag{2}$$

It follows from above that:

(a). if $c_{i,\sigma^*} > 0$ then $c_{-i,\sigma^*} = 0$. This in turn implies that type v_i^*'s expected payoff $U_i(v_i^*|\sigma^*) = 0$ and $x_{\sigma^*} = v_i^*$.

(b). $x_{\sigma^*} < v_i^*$ is equivalent to $c_{-i,\sigma^*} > 0$. It follows from above that if $x_{\sigma^*} < v_i^*$, then $c_{-i,\sigma^*} > 0$. If $c_{-i,\sigma^*} > 0$, then the probability of player 2 bidding zero is strictly positive in σ^*,[7] which implies $x_{\sigma^*} > \inf \operatorname{supp} \tilde{F}_2$ and

$$U_i(v_i^*|\sigma^*) = v_i^* - x_{\sigma^*} = v_i^* c_{-i,\sigma^*} > 0.$$

It follows that $x_{\sigma^*} < v_i^*$.

Remark 3. The following lemma concerns the expected payoff of type v_i in the all-pay auction in which player $-i$ believes that the common belief of the game is $\mathcal{G}(v_i^*, \tilde{F}_{-i})$ while player i secretly knows that his own type is v_i.

Lemma 3. *Given the above H_{i,σ^*}, for any type v_i: if $c_{-i,\sigma^*} = 0$ and $v_i \leq v_i^*$, then bidding zero is a best response to H_{-i,σ^*} and the expected payoff is $c_{-i,\sigma^*} v_i = 0$; if $v_i > v_i^*$, then the best response is to bid x_{σ^*} and thus the expected payoff is $v_i - x_{\sigma^*}$ or equivalently $v_i - v_i^*(1 - c_{-i,\sigma^*})$.*

Proof. Lemma 14 in Zheng (2019b). □

[7] It means that the lowest possible type must earn zero payoff in σ^*.

3 The bribing model

3.1 Peace implementability and security

It is obvious that in any peaceful equilibria there cannot be multiple bribes that are accepted with probability one. So any peaceful equilibrium is pooling.

Consider a peaceful equilibrium in which all types of player 1 pool the bribe at $\bar{b} \in [0, \underline{v}_1]$ which is accepted by all types of player 2. In the equilibrium, type v_1's payoff is $v_1 - \bar{b}$ whereas type v_2's payoff is $\bar{b}$. In such a peaceful equilibrium, there are two types of off-path deviations. On one hand, rejection is off the path. On the other hand, there are many unsent bribes.

We consider first player 2's rejection.

Suppose player 2 unexpectedly rejects the bribe $\bar{b}$. In the continuation auction, player 1's type distribution is the same as the prior. Because rejection is off the path, player 1 can hold arbitrary beliefs about player 2's type distribution.

It is not profitable for all types of player 2 to reject $\bar{b}$ if and only if the highest type of player 2, namely $\bar{v}_2$, earns an expected payoff lower than $\bar{b}$ because for any belief the expected payoff function of player 2 is increasing (by Lemma 1). Let the expected payoff of type v_2 in the continuation auctions following the rejection of player 2 be $U_2(v_2|\sigma_2)$ for a belief $\tilde{F}_2$. According to Zheng (2019b), for type $\bar{v}_2$, the highest possible $U_2(\bar{v}_2|\sigma_2)$ is achieved when player 1 believes $v_2 = \underline{v}_2$ and the lowest possible $U_2(\bar{v}_2|\sigma_2)$ is achieved when player 1 believes $v_2 = \bar{v}_2$. Let $\bar{\sigma}_2$ denote a BNE induced by belief $v_2 = \bar{v}_2$ and $\underline{\sigma}_2$ a BNE induced by belief $v_2 = \underline{v}_2$. So implementability of peace requires that $U_2(\bar{v}_2|\bar{\sigma}_2) \leq \bar{b}$ whereas security of peace requires that $U_2(\bar{v}_2|\underline{\sigma}_2) \leq \bar{b}$. Or equivalently, the former requires

$$\bar{v}_2 c_{1,\bar{\sigma}_2} \leq \bar{b}, \tag{3}$$

whereas the latter requires that

$$\bar{v}_2 - \underline{v}_2(1 - c_{1,\underline{\sigma}_2}) \leq \bar{b}. \tag{4}$$

where

$$c_{1,\bar{\sigma}_2} := \inf\{c_1 \in [0,1] : \bar{v}_2 \int_{c_1}^{1} \frac{1}{F_1^{-1}(s)} ds \leq 1\},$$

$$c_{1,\underline{\sigma}_2} := \inf\{c_1 \in [0,1] : \underline{v}_2 \int_{c_1}^{1} \frac{1}{F_1^{-1}(s)} ds \leq 1\}.$$

We now turn to player 1's deviation to off-path bribes. We first characterize the structure of player 2's replies (with a positive rejection probability) to the off-path bribes. When the on-path bribe is positive, then zero bribe is off-path and may have multiple consistent replies. The following result shows that for any consistent reply of player 2 induced by any off-path b there is an equivalent one with an interval rejection set.

Lemma 4. *For any off-path bribe b, for any given $\tilde{F}_1$, any consistent reply is equivalent to one with a rejection set $[a_{2,\sigma}(b), \overline{v_2}]$ for some $a_{2,\sigma}(b)$ which induces a BNE σ.*[8] *In σ,*

$$c_{2,\sigma} = 0.$$

Proof. See Appendix A.1. □

Surprisingly, we find that in contrast to Zheng (2019b), peace security is impossible in our model.

Theorem 1. *Peace is not securable.*

Proof. See Appendix A.2. □

In the proof we first show that if peace is securable, it can only be secured by a unique bribe, which is the highest possible expected payoff that type $\overline{v_2}$ earns in the continuation auction when she rejects the on-path bribe and is believed to be the lowest type. Then we show that if an off-path bribe $b \in (0, \overline{b})$ is rejected for the belief $v_1 = \overline{v_1}$, then in the continuation auction the expected payoff of any type of player 2 is no greater than $\overline{v_2} - \overline{v_1}$. It is this fact that renders peace security impossible because it turns out that player 1 can always deviate to some bribe lower than $\overline{b}$ and it should always be accepted by player 2.

We proceed to examine peace implementability.

Let $\pi_1(v_1|b, \tilde{F}_1)$ be the expected payoff of type v_1 from a deviation to an off-path bribe b when player 2's belief is $\tilde{F}_1$ which induces a continuation auction $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$.

Lemma 5. *For any off-path bribe b, for any $\tilde{F}_1$, any BNE σ of any induced $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$, $\pi_1(v_1|b, \tilde{F}_1)$ increases continuously at rates no greater than one.*

Proof. See Appendix A.3. □

[8] Although $a_{2,\sigma}(b)$ depends on b, to save notation, the argument is suppressed in the proofs.

The above result implies that implementability critically depends on the expected payoff of type $\underline{v}_1$ from deviations to all possible off-path bribes, because the equilibrium payoff of player 1 increases with type v_1 at a rate equal to one. That is, if it is not profitable for type $\underline{v}_1$ to deviate to some b for some belief $\tilde{F}_1$, then with the same belief it is not profitable for any type $v_1 > \underline{v}_1$ to deviate to the same b.

Hence, the examination of peace implementability amounts to searching for the belief $\tilde{F}_1$ that minimizes $\pi_1(\underline{v}_1|b,\tilde{F}_1)$ for each b. Peace is implementable only if the maximum of the minimized $\pi_1(\underline{v}_1|b,\tilde{F}_1)$ among all off-path b does not exceed the equilibrium payoff of type $\underline{v}_1$. That is, peace implementability requires

$$\max_b \min_{\tilde{F}_1} \pi_1(\underline{v}_1|b,\tilde{F}_1) \leq \underline{v}_1.$$

Lemma 6. *For any off-path bribe b, for any $\tilde{F}_1$, any BNE σ of any induced $\mathcal{G}(\tilde{F}_1,\tilde{F}_2)$,*

$$\pi_1(\underline{v}_1|b,\tilde{F}_1) = F_2(a_{2,\sigma}(b))(\underline{v}_1 - b). \tag{5}$$

Proof. See Appendix A.4. □

The structure of $\pi_1(\underline{v}_1|b,\tilde{F}_1)$ implies that for each b, $\pi_1(\underline{v}_1|b,\tilde{F}_1)$ is minimized by the lowest possible $a_{2,\sigma}(b)$ induced by some $\tilde{F}_1$. Before identifying such $a_{2,\sigma}(b)$, it is convenient to consider the lowest rejecting type of player 2, denoted by $a_{2,\underline{\sigma}}(b)$, when player 2's belief is $v_1 = \underline{v}_1$ and some BNE $\underline{\sigma}$ is induced. Below we give the conditions for $a_{2,\underline{\sigma}}(b) \leq \bar{v}_2$ and show that it is unique.

Let $\Phi_2(v_2|x)$ be the probability distribution of v_2 conditional on $v_2 \geq x$, i.e.,

$$\Phi_2(v_2|x) = \frac{F_2(v_2) - F_2(x)}{1 - F_2(x)},$$

and denote the inverse function by $\Phi_2^{-1}(\cdot|x)$. Let

$$\mathcal{I}_2(x) := \int_0^1 \frac{1}{\Phi_2^{-1}(s|x)} ds.$$

When x increases, $\Phi_2(v_2|x)$ becomes more first-order stochastic dominant. Stochastic dominance implies that $\mathcal{I}_2(x)$ is decreasing in x. It is also clear that $\mathcal{I}(\underline{v}_2) = \int_0^1 (F_2^{-1}(s))^{-1} ds$.

Lemma 7. *Consider an off-path bribe b. Suppose the induced belief $\tilde{F}_1$ is $v_1 = \underline{v}_1$. If $b \leq \bar{v}_2 - \underline{v}_1$, a consistent reply with a non-empty rejection set $[a_{2,\underline{\sigma}}(b), \bar{v}_2]$ exists and is unique; if $b > \bar{v}_2 - \underline{v}_1$, the rejection set is empty.*

When $b \leq \bar{v}_2 - \underline{v}_1$*,*

$$a_{2,\underline{\sigma}}(b) = \begin{cases} \underline{v}_2 & \text{if } 1 - \frac{b}{\underline{v}_2} > \underline{v}_1 \int_0^1 \frac{1}{F_2^{-1}(s)} ds \\ a_2 & \text{otherwise} \end{cases} \tag{6}$$

where a_2 *satisfies*

$$1 - \frac{b}{a_2} = \underline{v}_1 \mathcal{I}_2(a_2)$$

which admits a unique solution. Furthermore, $a_{2,\underline{\sigma}}(b)$ *is non-decreasing.*

Proof. See Appendix A.5. □

The following result shows that for each b, $\pi_1(\underline{v}_1 | b, \tilde{F}_1)$ is minimized by the belief $v_1 = \underline{v}_1$, or equivalently $a_{2,\underline{\sigma}}(b)$.

Lemma 8. *For any given off-path bribe* b*, among all possible* $\tilde{F}_1$*, any BNE* σ *of any induced* $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$*,* $x_{\underline{\sigma}} \leq x_\sigma$ *and* $a_{2,\underline{\sigma}}(b) \leq a_{2,\sigma}(b)$*.*

Proof. See Appendix A.6. □

We are now ready to state our second main result. Let

$$b^* \in \arg \max_{b \leq \max\{0, \bar{v}_2 - \underline{v}_1\}} \pi_1(\underline{v}_1 | b, \delta_{\underline{v}_1}) = F_2(a_{2,\underline{\sigma}}(b))(\underline{v}_1 - b),$$

where $a_{2,\underline{\sigma}}(b)$ is given by (6).[9]

Theorem 2. *If* $\bar{v}_2 \leq \underline{v}_1$*, then peace is implementable (through a zero bribe) if and only if*

$$c_{1,\bar{\sigma}_2} = 0.$$

If $\bar{v}_2 > \underline{v}_1$*, then peace is implementable if and only if*

$$\bar{v}_2 c_{1,\bar{\sigma}_2} + F_2(a_{2,\underline{\sigma}}(b^*))(\underline{v}_1 - b^*) \leq \underline{v}_1. \tag{7}$$

Furthermore, all peaceful equilibria are robust.[10]

Proof. See Appendix A.7. □

[9] Although in principle there could be multiple stationary points, a solution and the maximum can be solved by a standard approach.

[10] As noted in Zheng (2019b), the same result (the equivalence of robustness and implementability) obtains in his model.

Remark 4. With Lemma 7, it is straightforward to check whether the conditions for implementability is satisfied for given type distributions F_1 and F_2.

Remark 5. It follows from the proof of the result above that when (7) holds, any $\bar{b} \in [\bar{v}_2 c_{1,\bar{\sigma}_2}, \underline{v}_1 - F_2(a_{2,\underline{\sigma}}(b^*))(\underline{v}_1 - b^*)]$ can sustain a peaceful equilibrium. Of course, among all peaceful equilibria, the maximum payoff of player 1 is achieved in the one with $\bar{b} = \bar{v}_2 c_{1,\bar{\sigma}_2}$.

Corollary 1. *If $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds \leq 1$, then peace is implementable (at least in an equilibrium with $\bar{b} = 0$). If $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds > 1$ and $\underline{v}_1 = 0$, then peace is not implementable.*

Proof. See Appendix A.8. □

We now consider comparative statics for peace implementability. Zheng (2019b) shows that in his mediation model if either of the two players becomes stronger in the sense that the type distribution becomes more first-order stochastic dominant and the support remains unchanged, then peace implementability is preserved. It follows immediately from (7) that the same result in the case of player 1 obtains in our model for the same reason. (Corollary 1 is an example. If $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds \leq 1$ and thus peace is implementable and if $\hat{F}_1$ first-order stochastically dominates F_1, then $\bar{v}_2 \int_0^1 (\hat{F}_1^{-1}(s))^{-1} ds \leq \bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds \leq 1$ and peace implementability is preserved with $\hat{F}_1$.) In the case of player 2, a natural guess would be that the same result should obtain as well in our model because it seems intuitive that when player 2 becomes stronger, the lowest type of player 1 would earn a lower expected payoff from deviation and thus peace implementability should be preserved. However, the following example shows that the guess is not necessarily true and peace implementability may become impossible if player 2 becomes stronger.

Example 1. In this example we can focus on the second term of the left side of (7), namely $F_2(a_{2,\underline{\sigma}}(b^*))(\underline{v}_1 - b^*)$.[11] Suppose F_2 is uniformly distributed on $[0, 100]$ and $\underline{v}_1 = 30$. It is straightforward to obtain

$$\Phi_2^{-1}(s|x) = (100 - x)s + x \quad \text{and} \quad \int_0^1 \frac{1}{\Phi_2^{-1}(s|x)} ds = \frac{\log\left(\frac{x}{100}\right)}{x - 100}.$$

Numerical methods show that

$$b^* \approx 12.1762, a_{2,\underline{\sigma}}(b^*) \approx 26.5604, F_2(a_{2,\underline{\sigma}}(b^*))(\underline{v}_1 - b^*) \approx 4.73407.$$

With some suitable F_1, (7) holds with equality and peace is implementable only through b^*.

[11] $c_{1,\bar{\sigma}_2}$ in (7) depends only on F_1 and $\bar{v}_2$.

Suppose now that player 2's distribution becomes a different one but with the same support $[0,100]$,

$$\hat{F}_2(v_2) = \begin{cases} \frac{v_2^2}{3000} & 0 \le v_2 \le 30 \\ \frac{v_2}{100} & 30 \le v_2 \le 100. \end{cases}$$

It is straightforward to see that $\hat{F}_2$ first-order stochastically dominates F_2. By the same procedure as above,

$$\hat{b}^* \approx 13.4631, \hat{a}_{2,\underline{\sigma}}(\hat{b}^*) \approx 29.9416, \hat{F}_2(\hat{a}_{2,\underline{\sigma}}(\hat{b}^*))(\underline{v}_1 - \hat{b}^*) \approx 4.94177.$$

Thus, with the same F_1 as the above, (7) fails with $\hat{F}_2$ and peace is not implementable. The reason is that with $\hat{F}_2$, the probability mass concentrates more to the left of $v_2 = 30$. So it is advantageous for type $\underline{v}_1$ to offer the higher bribe $\hat{b}^*$ which leads to a higher probability of acceptance $\hat{F}_2(\hat{a}_{2,\underline{\sigma}}(\hat{b}^*)) \approx 0.298833 > 0.265604 \approx F_2(a_{2,\underline{\sigma}}(b^*))$. And acceptance helps avoid competing with player 2 in the continuation auction in which type $\underline{v}_1$ earns zero expected payoff. It turns out that this effect could outweigh the effect from the higher payment in the case of acceptance and thus yield a higher expected payoff for type $\underline{v}_1$ off the path. This renders peace implementability impossible.

3.2 Non-peaceful equilibria

We now consider non-peaceful equilibria. In any equilibrium, if an on-path bribe is accepted with probability one, then any other on-path bribes must be rejected with a positive probability.[12] Lemma 4 also implies that for any on-path bribe b the rejection set can be described by an interval $[a_{2,\sigma}(b), \bar{v}_2]$ where σ is a BNE of the continuation auction.

We first rule out decreasing equilibria and non-monotonic equilibria.

Lemma 9. *In any equilibrium, the bribing function is non-decreasing everywhere.*

Proof. See Appendix A.9. □

We next consider equilibria with continuously increasing segments.

Lemma 10. *There exist no equilibria in which the equilibrium bribing function is continuously increasing over some type interval.*

Proof. See Appendix A.10. □

[12] If there is a different bribe b' offered by a different type v_1' and accepted with probability one, then the lower bribe is preferred by both types.

The result above excludes possibility of equilibria with separating segment(s), which is reminiscent of the result in Rachmilevitch (2013). In particular, the result implies that there exist no separating equilibria.

We proceed to examine the possibility of other types of equilibria and obtain the following result which shows that in any equilibrium the number of on-path bribes is at most two.

Theorem 3. *Suppose there exists a non-peaceful equilibrium. Then either it is a pooling equilibrium, or in the equilibrium, there are two on-path bribes, b_l and b_h with $b_l < b_h$. In the latter case, for some $v_1^* \in (\underline{v}_1, \bar{v}_1)$, bribe b_l is offered by types $v_1 \in [\underline{v}_1, v_1^*\rangle$ and rejected with a positive probability, and b_h is offered by $v_1 \in \langle v_1^*, \bar{v}_1]$ and accepted with probability one.*[13]

Proof. See Appendix A.11. □

4 Discussion: the requesting model

In this section, we consider the opposite scenario in which instead of offering a bribe, player 1 commits to the bargaining protocol of requesting a bribe. Again, peace is not implementable in any separating equilibria for similar reasons in the bribing model. Below we focus on robust peaceful equilibria (which can only be pooling).

Suppose that a peaceful equilibrium exists, with the on-path request denoted by $\bar{r}$ which cannot be higher than $\underline{v}_2$.

Consider an off-path request r. Similar to the bribing model, for some belief, if it is optimal for some type v_2 to reject r, then it is also optimal for any type $v_2' < v_2$ to reject the request.[14] So if the rejection set is non-empty, then it is an interval $[\underline{v}_2, \alpha_{2,\sigma}(r)]$ for some $\alpha_{2,\sigma}(r) \leq \bar{v}_2$. Clearly, it is optimal for all types $v_2 < r$ to reject the request r and thus $\alpha_{2,\sigma}(r) \geq \min\{\bar{v}_2, r\}$.

For any given belief $\tilde{F}_1$ and any BNE σ of the induced continuation auction, the expected payoff of type $\bar{v}_1$ from an off-path request r is

$$\pi_1(\bar{v}_1|r, \tilde{F}_1) = F_2(\alpha_{2,\sigma}(r))(\bar{v}_1 - x_\sigma) + (1 - F_2(\alpha_{2,\sigma}(r)))r. \tag{8}$$

If the consistent reply in σ is partial rejection (i.e., $\alpha_{2,\sigma}(r) < \bar{v}_2$), then type $\alpha_{2,\sigma}(r)$ must be indifferent between rejecting and paying r. In that case, because type $\alpha_{2,\sigma}(r)$ is the highest type of the rejection set, she must bid x_σ in any BNE σ of the continuation auction and win with probability one and thus $x_\sigma = r$. if the consistent reply is full rejection (i.e., $\alpha_{2,\sigma}(r) = \bar{v}_2$), then $x_\sigma \leq r$.

[13] $[\underline{v}_1, v_1^*\rangle$ means $[\underline{v}_1, v_1^*]$ or $[\underline{v}_1, v_1^*)$. Similarly, $\langle v_1^*, \bar{v}_1]$ means $[v_1^*, \bar{v}_1]$ or $(v_1^*, \bar{v}_1]$.

[14] For any given belief, the expected payoff of player 2 is increasing at a rate lower than one.

Lemma 11. *If $\bar{v}_1 > 2\underline{v}_2$, then there exist no peaceful equilibria; if $\bar{v}_1 \leq 2\underline{v}_2$, then in any peaceful equilibrium, $\bar{r} \geq \bar{v}_1/2$.*

Proof. See Appendix B.1. □

Let $\Psi_2(v_2|x) := F_2(v_2)/F_2(x)$ be the probability distribution of v_2 conditional on $v_2 \leq x$ and let $\Psi_2^{-1}(\cdot|x)$ be the inverse function. Clearly $\int_0^1 (\Psi_2^{-1}(s|\bar{v}_2))^{-1} ds = \int_0^1 (F_2^{-1}(s))^{-1} ds$.

Lemma 12. *There exist no robust peaceful equilibria if $\bar{v}_1 \leq \underline{v}_2$.*

Proof. See Appendix B.2. □

It follows from Lemma 11 and 12 that a robust peaceful equilibrium exists only if $\underline{v}_2 < \bar{v}_1 \leq 2\underline{v}_2$. We assume it is satisfied for the analysis below.

Consider $\mathcal{G}(\delta_{\bar{v}_1}, F_2)$ and denote a BNE by $\bar{\sigma}^*$. Let $c_{2,\bar{\sigma}^*}$ be the value that (uniquely) solves

$$\bar{v}_1 \int_{c_{2,\bar{\sigma}^*}}^1 \left(F_2^{-1}(s)\right)^{-1} ds = 1$$

if $\bar{v}_1 \int_0^1 (F_2^{-1}(s))^{-1} ds > 1$ and equals zero otherwise. Let the highest bid in $\bar{\sigma}^*$ be $x_{\bar{\sigma}^*}$. Then by Lemma 2,

$$x_{\bar{\sigma}^*} = \bar{v}_1(1 - c_{2,\bar{\sigma}^*}).$$

Let $\alpha_{2,\bar{\sigma}}(r)$ be (uniquely) given by

$$\bar{v}_1 \int_{c_{2,\bar{\sigma}}}^1 \left(\Psi_2^{-1}(s|\alpha_{2,\bar{\sigma}}(r))\right)^{-1} ds = 1 \tag{9}$$

where $c_{2,\bar{\sigma}}$ is given by $r = \bar{v}_1(1 - c_{2,\bar{\sigma}})$, if (9) admits an $\alpha_{2,\bar{\sigma}}(r) \in [\underline{v}_2, \bar{v}_2]$.[15] And let

$$r^* \in \arg\max_{r \in [\underline{v}_2, x_{\bar{\sigma}^*}]} F_2(\alpha_{2,\bar{\sigma}}(r))(\bar{v}_1 - r) + (1 - F_2(\alpha_{2,\bar{\sigma}}(r)))r. \tag{10}$$

The following result characterizes the necessary and sufficient conditions for the existence of a robust peaceful equilibrium.

Theorem 4. *In the requesting model, there exists a robust peaceful equilibrium if and only if the following conditions are satisfied:*

(a). $\underline{v}_2 < \bar{v}_1 \leq 2\underline{v}_2$;

[15] When $\alpha_{2,\bar{\sigma}}(r)$ decreases, $\Psi_2(\cdot|\alpha_{2,\bar{\sigma}}(r))$ is more first-order stochastic dominant. Stochastic dominance implies that the integral $\int_{c_{2,\bar{\sigma}}}^1 \left(\Psi_2^{-1}(s|\alpha_{2,\bar{\sigma}}(r))\right)^{-1} ds$ is increasing in $\alpha_{2,\bar{\sigma}}(r)$. Thus if for a given $c_{2,\bar{\sigma}}$, (9) admits an $\alpha_{2,\bar{\sigma}}(r) \in [\underline{v}_2, \bar{v}_2]$, then it is unique.

(b). $\underline{v}_2 c_{1,\underline{\sigma}_2} = 0$;

(c). $\underline{v}_2 = F_2(\alpha_{2,\bar{\sigma}}(r^*))(\bar{v}_1 - r^*) + (1 - F_2(\alpha_{2,\bar{\sigma}}(r^*)))r^*$.

In any robust peaceful equilibrium, $\bar{r} = \underline{v}_2$.

Proof. See Appendix B.3. □

The intuition for $\bar{r} = \underline{v}_2$ is the following. In any robust peaceful equilibrium with D1, for any off-path request, the only reasonable belief is $v_1 = \bar{v}_1$, because the expected payoff of type v_1 from the off-path deviation is increasing. From above, $\bar{v}_1 > \underline{v}_2$ in any robust peaceful equilibrium. If type $\bar{v}_1$ deviates to any off-path request $r < \underline{v}_2$, then it is accepted by all types v_2. To see this, suppose to the contrary the rejection set is non-empty. Recall that the rejection set consists of the low types of player 2, in particular, type $\underline{v}_2$. In the continuation auction, if type $\underline{v}_2$ wins with a positive probability, then type $\bar{v}_1$ earns zero payoff and the highest bid is $\bar{v}_1$ which is the bid of the highest type of player 2 in the auction, namely type $\alpha_{2,\bar{\sigma}}$. Since $r < \underline{v}_2 < \bar{v}_1$, type $\alpha_{2,\bar{\sigma}}$ would rather have accepted r. If type $\underline{v}_2$ wins with zero probability and thus earns zero payoff, it is lower than the payoff from accepting r. So the only consistent reply of player 2 is to accept $r < \underline{v}_2$ for all types v_2. So if the equilibrium request $\bar{r} < \underline{v}_2$, then type $\bar{v}_1$ can always deviate to some $r \in (\bar{r}, \underline{v}_2)$ and earn a higher payoff. Since the equilibrium request cannot exceed $\underline{v}_2$, it can only be $\underline{v}_2$.

We proceed to examine peace security.

Lemma 13. *For any given off-path r, for any possible beliefs and any BNE σ of the induced continuation auction,*

$$\alpha_{2,\underline{\sigma}} \geq \alpha_{2,\sigma} \geq \alpha_{2,\bar{\sigma}} \quad \text{and} \quad x_{\underline{\sigma}} \leq x_{\sigma} \leq x_{\bar{\sigma}}. \tag{11}$$

Proof. See Appendix B.4. □

Remark 6. The inequalities in (11) imply

(a). if full rejection is the consistent reply to r for the belief $v_1 = \bar{v}_1$, then it is also the consistent reply for any belief;

(b). if full rejection is the consistent reply to r for some belief, then it is also the consistent reply for the belief $v_1 = \underline{v}_1$;

(c). if full acceptance is the consistent reply to r for belief $v_1 = \underline{v}_1$, then it is also the consistent reply to r for any belief;

(d). if full acceptance is the consistent reply to r for some belief, then it is also the consistent reply to r for the belief $v_1 = \bar{v}_1$.

The following results characterize the necessary and sufficient conditions for peace security.

Theorem 5. *Suppose that there exists a robust peaceful equilibrium in the requesting model. If $\underline{v}_1 > \underline{v}_2$, then peace is securable if and only if*

$$\underline{v}_2 c_{1,\bar{\sigma}_2} = 0. \tag{12}$$

If $\underline{v}_1 \leq \underline{v}_2$, then peace is securable if and only if in addition to (12),

$$\bar{v}_1 - \underline{v}_1 \leq \underline{v}_2.$$

Proof. See Appendix B.5. □

Interestingly, peace security does not require more restrictive conditions on player 1's primitive than the ones for a robust peaceful equilibrium if $\underline{v}_1 > \underline{v}_2$ while it does require some more restrictive conditions on player 1's primitive if $\underline{v}_1 \leq \underline{v}_2$. This is because in the latter case, if $\bar{v}_1 - \underline{v}_1 > \underline{v}_2$, then it could be possible for the highest type of player 1 to trick player 2 with the lowest belief ($v_1 = \underline{v}_1$) to reject the request and submit a low bid in the continuation auction induced by a high (or low) enough request. But in the former case, the lowest type of player 1 is high enough and thus it is meaningless to trick player 2 with the lowest belief.

5 Conclusion

We study a simple model of conflict preemption in which one party actively bargains with the other one through a take-it-or-leave-it offer of a payment. We adopt the notions for various degrees of peace prospects in the received literature. We find that peace security is impossible in our model, a result in contrast to the one in the mediation model in Zheng (2019b). Such a qualitatively different result is due to the different strategic considerations of the briber when taking an off-path action. The briber's intention is no longer to pretend to be weak and trick the opponent in the continuation auction (as in the mediation model), but rather to pretend strong and force the opponent to accept a lower bribe.

We characterize the necessary and sufficient conditions for peace implementability. With the conditions, we find that unlike in Zheng (2019b), peace implementability may be dismissed if the receiver becomes stronger in our model. The reason is that with a stronger receiver, it could be more advantageous to offer a higher off-path bribe. The higher bribe leads to a significantly lower probability of conflict and a modestly higher payment in the case of acceptance. And the overall effect could be a higher expected payoff of the weakest type of the briber which renders peace implementability impossible.

For non-peaceful equilibria, we find that there exist no separating equilibria in our model. This result echoes with the findings in the bribing model of Rachmilevitch (2013) for first-price auction, but in contrast to Esö and Schummer (2004). We also show that in any non-peaceful equilibrium, the number of on-path bribes is at most two, and if it is two, then the higher one is accepted with certainty and the lower one is rejected by a positive probability.

We also consider a requesting model and characterize the necessary and sufficient conditions for the existence of robust peaceful equilibria. We find that all such equilibria share the identical on-path request which is the lowest possible valuation of the player paying it. Finally, we find that contrary to the bribing model, peace security is possible in the requesting model.

In comparison, each of the bribing model and the requesting model has its own pros and cons. In the bribing model, peace typically can be implemented robustly through a continuum of bribes, but it is not securable; in the requesting model, peace can be robustly implemented and secured, but only through a single request.

Appendices

A Proofs for the bribing model

A.1 Proof of Lemma 4

It is convenient to consider the off-path zero bribe first when the zero bribe is not on the path, because there may be multiple consistent replies to the off-path zero bribe. Specifically, it may be that in some consistent reply, there exist some gaps in some rejection set V_2: some types at the bottom of V_2 reject the bribe and bid zero (and earn zero payoff) in the continuation auction, whereas some higher types accept the zero bribe; all these types earn the same zero payoff and thus are indifferent between accepting the zero bribe and bidding zero in the continuation auction; therefore there can be multiple consistent replies to the zero bribe even for the same belief $\tilde{F}_1$.[16] We show in the following claim that we can focus on a particular type of consistent replies, the *effective* replies. In the effective replies, the rejection set is an interval (without any gap) and each rejecting type earns a positive payoff in any BNE σ of the induced continuation auction except the lowest rejecting type.

Claim 1. Given the off-path zero bribe, suppose for some belief $\tilde{F}_1$ there exists a non-empty consistent reply associated with a BNE σ. Then there exists an equivalent consistent reply: for some $a_{2,\sigma}(0)$, any type $v_2 < a_{2,\sigma}(0)$ accepts the zero bribe and any type v_2 in $[a_{2,\sigma}(0), \bar{v}_2]$ rejects the zero bribe and uses the same strategy as in σ.

Proof. Consider the off-path zero bribe and some consistent reply with the induced $\mathcal{G}$ and a BNE σ by some belief $\tilde{F}_1$. If type v_2 in the rejection set V_2 earns a positive expected payoff in σ, then it is optimal for any type $v_2' > v_2$ to reject the bribe and earn a positive expected payoff in the auction.

[17] Let V_{2+} be the set of types earning a positive payoff and let $a_{2,\sigma'} := \inf V_{2+}$. So any type in $V_{2-} := \{v_2 | v_2 \leq a_{2,\sigma'}, v_2 \in V_2\}$ earns zero payoff in σ. On the other hand, because any type $v_2 \in V_{2-}$ is indifferent between accepting the zero bribe and bidding zero in the auction, there exists an equivalent consistent reply to $\tilde{F}_1$ in which all types in $[a_{2,\sigma'}, \bar{v}_2]$ reject the zero bribe and the other types accept the zero bribe. In the continuation auction induced by the equivalent consistent reply, each type of each player uses the same strategy as in σ.

[16] For positive off-path bribes, this kind of multiplicity of consistent reply does not arise. We show the uniqueness in Lemma 7.

[17] Type v_2' at least can use the same strategy as type v_2's and thus win with the same probability and thus earn a higher expected payoff.

To see the equivalence, let the equivalent continuation game be denoted by $\mathcal{G}'$ and $V_2' = [a_{2,\sigma'}, \bar{v}_2]$. Given the invariant behavior of types $v_2 \in V_2'$, if it is a best response for some type v_1 to bid β in $\mathcal{G}$, then β is still a best response for type v_1 in $\mathcal{G}'$. To see this, let $\hat{H}_2 = P(v_2 < a_{2,\sigma'} | v_2 \in V_2)$. If type v_1 bids β in σ and the win probability is $H_{1,\sigma}(\beta)$, the expected payoff from the auction is

$$U_1(v_1, \beta | \sigma) = H_{1,\sigma}(\beta) v_1 - \beta.$$

Given the invariant behavior of types $v_2 \in V_2'$, if type v_1 bids β in σ', the win probability is reduced to $H_{1,\sigma}(\beta) - \hat{H}_2$ and thus the expected payoff from the auction is

$$U_1(v_1, \beta | \sigma') = (H_{1,\sigma}(\beta) - \hat{H}_2) v_1 - \beta.$$

Clearly, β remains a best response of type v_1 in σ' because $\hat{H}_2 v_1$ is a constant for type v_1. So $H_{1,\sigma}$ remains the best bid distribution of player 1 in σ'. Then, if player 1's bid distribution is $H_{1,\sigma}$ in σ', it is optimal for types $v_2 \in V_2'$ to use the same strategy as in σ. It is clear now that the new consistent reply is equivalent to the old one. $\square$

Now we discuss zero bribe and non-zero bribes separately.

If $b > 0$, then the rejection set V_2 is an interval $[a_{2,\sigma}, \bar{v}_2]$ for any belief $\tilde{F}_1$.[18] In particular, because type $a_{2,\sigma}$ earns a positive payoff in σ, the bid distribution $H_{2,\sigma}$ has no atom at zero, i.e., $c_{2,\sigma} = 0$.

If $b = 0$, then for any belief $\tilde{F}_1$ by the claim above we can focus on the effective replies with an interval rejection set $[a_{2,\sigma}, \bar{v}_2]$ as well. It follows that for any σ, $c_{2,\sigma} = 0$. To see this, suppose to the contrary, $c_{2,\sigma} > 0$ and thus $H_{2,\sigma}$ has an atom at zero. Then it follows that $c_{1,\sigma} = 0$ and thus for any type v_2 bidding zero yields zero win probability and thus zero payoff. So if type v_2 earns a positive payoff in σ, then zero bid cannot be the optimal bid. Since each type $v_2 > a_{2,\sigma}$ earns a positive payoff, only type $v_2 = a_{2,\sigma}$ bids zero. But then this implies that $c_{2,\sigma} = 0$.

A.2 Proof of Theorem 1

First, peace can only be secured by a unique bribe. Suppose two bribes can secure peace, say $\bar{b}$ and b' with $b' > \bar{b}$. Consider a peaceful equilibrium with b' and another peaceful equilibrium with $\bar{b}$. Denote the former equilibrium by P' and the latter by $\bar{P}$. By security, in equilibrium $\bar{P}$ it is not profitable for any type of player 2 to reject the on-path bribe $\bar{b}$ for any belief about player

[18] With its own equilibrium strategy, the lowest type in the rejecting type set must win the object with a positive probability and earn an expected payoff equal to b. A higher type can use the same strategy and thus win with the same probability and thus earn a higher expected payoff.

2's type distribution (should player 2 reject $\bar{b}$). That is, for any v_2 and for any σ_2 induced by any $\tilde{F}_2$, $U_2(v_2|\sigma_2) \leq \bar{b}$. Consider an off-path deviation of player 1 to $b \in (\bar{b}, b')$ in equilibrium P'. Then

$$U_2(v_2|\sigma_2) \leq \bar{b} < b.$$

So upon receiving b, if the belief of player 2 is $v_1 \sim F_1$, then any type $v_2 < \bar{v}_2$ should accept b. Thus player 1 can profitably deviate to b for the belief $v_1 \sim F_1$ and bribe b' can not secure peace.

Next, we show that *the only possible candidate peaceful bribe for security is* $\bar{b} = U_2(\bar{v}_2|\underline{\sigma}_2)$. To see this, suppose peace is secured by a peaceful bribe $b' > U_2(\bar{v}_2|\underline{\sigma}_2)$. In a peaceful equilibrium with such b', $b \in (\bar{b}, b')$ is an off-path deviation. According to Zheng (2019b), for any $\tilde{F}_2$, $U_2(\bar{v}_2|\underline{\sigma}_2) \geq U_2(\bar{v}_2|\sigma_2)$, or equivalently $\bar{b} \geq U_2(\bar{v}_2|\sigma_2)$. Thus, by the similar arguments above, upon receiving b, because, given the belief $v_1 \sim F_1$, for any belief $\tilde{F}_2$ and any BNE σ_2 in the continuation auction, b should be accepted with probability one and thus b is a profitable deviation for player 1 for the belief $(F_1, \tilde{F}_2)$. This invalidates security by b'.

So next *suppose peace is secured by* $\bar{b} = U_2(\bar{v}_2|\underline{\sigma}_2)$. It follows then that $\underline{v}_1 \geq \bar{b}$ and $\bar{v}_1 > \bar{b}$. Observe also that $\bar{b} = U_2(\bar{v}_2|\underline{\sigma}_2) > 0$ and thus there always exist some off-path bribes $b < \bar{b}$.

Claim 2. Consider an off-path bribe $b \in (0, \bar{b})$. If it is rejected for the belief $v_1 = \bar{v}_1$, then in the continuation auction the expected payoff of any type of player 2 is no greater than $\bar{v}_2 - \bar{v}_1$.

Proof. Suppose for the belief $v_1 = \bar{v}_1$ some type $v_2 \in [a_{2,\bar{\sigma}}, \bar{v}_2]$ rejects it for some $a_{2,\bar{\sigma}}$. Then in any BNE $\bar{\sigma}$ of the continuation auction $\mathcal{G}(\delta_{\bar{v}_1}, \tilde{F}_2(a_{2,\bar{\sigma}}, \bar{v}_2))$, type $a_{2,\bar{\sigma}}$ must earn a positive payoff (because $b > 0$) and therefore $c_{1,\bar{\sigma}} > 0$. It follows that type $\bar{v}_1$ earns zero expected payoff and the highest bid $x_{\bar{\sigma}} = \bar{v}_1$. Type $\bar{v}_2$ must bid $\bar{v}_1$ and earns exactly $\bar{v}_2 - \bar{v}_1$ in $\mathcal{G}(\delta_{\bar{v}_1}, \tilde{F}_2(a_{2,\bar{\sigma}}, \bar{v}_2))$. Therefore, any type v_2 earns an expected payoff no greater than $\bar{v}_2 - \bar{v}_1$. □

It is clear from (4) that security requires $\bar{v}_2 - \underline{v}_2 \leq \bar{b}$. It follows that *security requires* $\bar{v}_1 \leq \underline{v}_2$. To see this, suppose $\bar{v}_1 > \underline{v}_2$, which implies $\bar{v}_2 - \bar{v}_1 < \bar{v}_2 - \underline{v}_2 \leq \bar{b}$. Then in some peaceful equilibrium player 1 can deviate to some $b \in (\max\{\bar{v}_2 - \bar{v}_1, 0\}, \bar{b})$. Security requires that b must lead to a positive probability of rejection for any belief; otherwise b is a profitable deviation for some belief and peace is not secured. In particular, security requires that for the belief $v_1 = \bar{v}_1$, type $\bar{v}_2$ prefers bidding competitively in game $\mathcal{G}(\delta_{\bar{v}_1}, \tilde{F}_2(a_{2,\bar{\sigma}}, \bar{v}_2))$ to accepting b for some $a_{2,\bar{\sigma}}$. By Claim 2 it is not profitable for any type of player 2 to reject b for the belief $v_1 = \bar{v}_1$, a contradiction.

Consider $\mathcal{G}(F_1, \delta_{\underline{v}_2})$. The fact that $\bar{v}_1 \leq \underline{v}_2$ implies that the highest bid in $\mathcal{G}(F_1, \delta_{\underline{v}_2})$ cannot exceed $\bar{v}_1$. It follows then that $U(\bar{v}_2|\underline{\sigma}_2) \geq \bar{v}_2 - \bar{v}_1$ or equivalently $\bar{v}_2 - \bar{v}_1 \leq U(\bar{v}_2|\underline{\sigma}_2) = \bar{b}$. But in fact, *security requires* $\bar{v}_2 - \bar{v}_1 = U(\bar{v}_2|\underline{\sigma}_2) = \bar{b}$. Suppose not, i.e., $\bar{v}_2 - \bar{v}_1 < U(\bar{v}_2|\underline{\sigma}_2) = \bar{b}$. Then player 1 can deviate to some $b \in (\bar{v}_2 - \bar{v}_1, \bar{b})$, which by Claim 2, for the belief $v_1 = \bar{v}_1$, is accepted with probability one, and thus the bribe b is a profitable deviation, a contradiction to security.

Finally, consider again $\mathcal{G}(F_1, \delta_{\underline{v}_2})$. Given the strategies of player 1 in any BNE $\underline{\sigma}_2$ of $\mathcal{G}(F_1, \delta_{\underline{v}_2})$, it is optimal for type $\bar{v}_2$ to bid the highest bid $x_{\underline{\sigma}_2}$. Thus the fact $\bar{v}_2 - \bar{v}_1 = U(\bar{v}_2|\underline{\sigma}_2)$ implies that in $\mathcal{G}(F_1, \delta_{\underline{v}_2})$, $x_{\underline{\sigma}_2} = \bar{v}_1$ and thus type $\bar{v}_1$ earns zero payoff, which is impossible.[19] This completes the proof.

A.3 Proof of Lemma 5

From Lemma 1, for any type $v_1 \in [\underline{v}_1, \bar{v}_1]$, the expected payoff in any given BNE σ of any continuation auction $\mathcal{G}(\tilde{F}_1, \tilde{F}_2)$ is $U_1(v_1|b, \tilde{F}_1) = \max_{\beta \in R_+} H_{2,\sigma}(\beta)v_1 - \beta$. Since $U_1(v_1|b, \tilde{F}_1)$ is the maximum of a family of affine functions, $U_1(v_1|b, \tilde{F}_1)$ is convex and thus absolutely continuous and differentiable almost everywhere. The expected payoff of any type $v_1 \in V_1$ from the deviation is

$$\pi_1(v_1|b, \tilde{F}_1) = F_2(a_{2,\sigma}(b))v_1 + (1 - F_2(a_{2,\sigma}(b)))(\max_{\beta \in R_+} H_{2,\sigma}(\beta)v_1 - \beta),$$

which is also absolutely continuous and differentiable everywhere.

Since $H_{2,\sigma} \leq 1$, whenever $\pi_1(v_1|b, \tilde{F}_1)$ is differentiable, the slope is not higher than one. At those non-differentiable points, the left and right derivatives of $\pi_1(v_1|b, \tilde{F}_1)$ are not higher than one. Thus, $\pi_1(v_1|b, \tilde{F}_1)$ increases continuously at rates no greater than one.

A.4 Proof of Lemma 6

If the rejection set is empty, then the expected payoff of player 1 from the deviation is $F_2(\bar{v}_2)(\underline{v}_1 - b) = \underline{v}_1 - b$. So below we assume non-empty rejection sets.

From above, for any b and $\tilde{F}_1$, $c_{2,\sigma} = 0$.

For a given b, suppose player 2 holds a belief $\tilde{F}_1$ with a support V_1. In the auction, any type v_1 must be indifferent among any positive bid β_1 in type v_1's bid support. This implies that for any such β_1,

$$H'_{2,\sigma}(\beta_1)v_1 - 1 = 0,$$

[19] In any BNE σ of $\mathcal{G}(F_1, \delta_{\underline{v}_2})$, if the strategy of some type $v_1 < \bar{v}_1$ wins a positive probability, then type $\bar{v}_1$ can use the same strategy and then earns a positive payoff. So in σ, any type $v_1 < \bar{v}_1$ must bid zero. It follows that the best response of type $\underline{v}_2$ is to bid 0^+. But then type $\bar{v}_1$ can win for sure by bidding 0^{++} and earn a positive payoff.

which in turn implies that $H'_{2,\sigma}(\beta_1)\underline{v}_1 - 1 < 0$ if $\underline{v}_1 \notin V_1$. So if $\underline{v}_1 \notin V_1$, it is optimal for type $\underline{v}_1$ to bid zero and earn $c_{2,\sigma}\underline{v}_1 = 0$ in the auction. Suppose now $\underline{v}_1 \in V_1$. If type $\underline{v}_1$ bids zero, then again he wins zero payoff because $c_{2,\sigma} = 0$. If supp $\sigma(\cdot, \underline{v}_1)$ is a non-degenerate interval $[0, x]$, then type $\underline{v}_1$ must be indifferent between any bids in $(0, x]$; but then type $\underline{v}_1$'s expected payoff is equal to $\lim_{\beta_1 \to 0} H_{2,\sigma}(\beta_1)\underline{v}_1 - \beta_1 = c_{2,\sigma}\underline{v}_1$, which is zero because $c_{2,\sigma} = 0$. This completes the proof.

A.5 Proof of Lemma 7

Suppose for some off-path b, the belief $v_1 = \underline{v}_1$ induces a consistent reply with a non-empty rejection set $[a_{2,\underline{\sigma}}, \bar{v}_2]$.

In the continuation auction $\mathcal{G}(\delta_{\underline{v}_1}, F_2(v_2|v_2 \geq a_{2,\underline{\sigma}}))$, $c_{2,\underline{\sigma}} = 0$ by Lemma 4. By Lemma 2, the boundary condition $H_{1,\underline{\sigma}}(x_{\underline{\sigma}}) = 1$ applying to (1) yields

$$1 - c_{1,\underline{\sigma}} = \underline{v}_1 \int_0^1 \frac{1}{\Phi_2^{-1}(s|a_{2,\underline{\sigma}})} ds = \underline{v}_1 \mathcal{I}_2(a_{2,\underline{\sigma}}). \tag{13}$$

Optimality of rejection for type $a_{2,\underline{\sigma}}$ requires that $U_2(a_{2,\underline{\sigma}}|b, \delta_{\underline{v}_2}) = c_{1,\underline{\sigma}} a_{2,\underline{\sigma}} \geq b$. So (13) and $1 - \frac{b}{a_{2,\underline{\sigma}}} \geq 1 - c_{1,\underline{\sigma}}$ imply

$$1 - \frac{b}{a_{2,\underline{\sigma}}} \geq \underline{v}_1 \mathcal{I}_2(a_{2,\underline{\sigma}}). \tag{14}$$

Observe that if $a_{2,\underline{\sigma}} = b < \bar{v}_2$, then $a_{2,\underline{\sigma}}$ is given by the equality of of (14) because the LHS is zero while the RHS is nonnegative. So if $a_{2,\underline{\sigma}} \in (\underline{v}_2, \bar{v}_2)$, then (14) holds with equality, which pins down the value of $a_{2,\underline{\sigma}}$. The LHS of (14) is an increasing function of $a_{2,\underline{\sigma}}$, which increases from $1 - \frac{b}{\underline{v}_2}$ to $1 - \frac{b}{\bar{v}_2}$ when $a_{2,\underline{\sigma}}$ increases from $\underline{v}_2$ to $\bar{v}_2$. When $a_{2,\underline{\sigma}}$ increases, $\Phi_2(v_2|a_{2,\underline{\sigma}})$ is more first-order stochastic dominant. Stochastic dominance implies that the integral $\int_0^1 \frac{1}{\Phi_2^{-1}(s|a_{2,\underline{\sigma}})} ds$ is decreasing in $a_{2,\underline{\sigma}}$ and it decreases from $\mathcal{I}_2(\underline{v}_2)$ to $\frac{1}{\bar{v}_2}$ when $a_{2,\underline{\sigma}}$ increases from $\underline{v}_2$ to $\bar{v}_2$. That is, the RHS of equation (14) decreases from $\underline{v}_1 \mathcal{I}_2(\underline{v}_2)$ to $\frac{\underline{v}_1}{\bar{v}_2}$. Thus, equality of (14) admits an solution $a_{2,\underline{\sigma}} \in (\underline{v}_2, \bar{v}_2)$ if and only if $\frac{\underline{v}_1}{\bar{v}_2} \leq 1 - \frac{b}{\bar{v}_2}$ (or equivalently $b \leq \bar{v}_2 - \underline{v}_1$) and $1 - \frac{b}{\underline{v}_2} \leq \underline{v}_1 \mathcal{I}_2(\underline{v}_2)$. And the solution is unique, given by equality of (14).[20]

On the other hand, if $b \leq \bar{v}_2 - \underline{v}_1$ and $1 - \frac{b}{\underline{v}_2} > \underline{v}_1 \mathcal{I}_2(\underline{v}_2)$, then $a_{2,\underline{\sigma}} = \underline{v}_2$.

Finally, if $b > \bar{v}_2 - \underline{v}_1$, then the rejection set is empty because from above a non-empty rejection set requires $b \leq \bar{v}_2 - \underline{v}_1$. Since $\mathcal{I}_2(\underline{v}_2) = \int_0^1 \frac{1}{F_2^{-1}(s)} ds$, this completes the proof.

[20]The uniqueness is due to the fact that $\frac{\underline{v}_1}{\bar{v}_2} \leq 1 - \frac{b}{\bar{v}_2}$ and $1 - \frac{b}{\underline{v}_2} \leq \underline{v}_1 \mathcal{I}_2(\underline{v}_2)$ can not hold with equality at the same time.

A.6 Proof of Lemma 8

We first consider non-empty rejection sets.

Suppose to the contrary, $a_{2,\underline{\sigma}} > a_{2,\sigma}$.

Given an off-path bribe b, consider $\underline{\sigma}$ and some σ. We note that $a_{2,\underline{\sigma}} > a_{2,\sigma}$ implies

$$c_{1,\underline{\sigma}} < c_{1,\sigma}. \tag{15}$$

If both $a_{2,\underline{\sigma}}$ and $a_{2,\sigma}$ are in the interior of the support of v_2, both types are indifferent between acceptance and rejection. So $a_{2,\underline{\sigma}}c_{1,\underline{\sigma}} = b = a_{2,\sigma}c_{1,\sigma}$. Thus $a_{2,\underline{\sigma}} > a_{2,\sigma}$ implies (15). If $a_{2,\sigma}$ is not in the interior, namely $a_{2,\sigma} = \underline{v}_2$, then type $a_{2,\underline{\sigma}} > \underline{v}_2$ and still is indifferent between acceptance and rejection in $\underline{\sigma}$ and thus $a_{2,\underline{\sigma}}c_{1,\underline{\sigma}} = b$. On the other hand, because type $a_{2,\sigma}$ rejects b in σ, he must earn no less than b in σ and thus $a_{2,\sigma}c_{1,\sigma} \geq b$. Therefore, the supposition of $a_{2,\underline{\sigma}} > a_{2,\sigma}$ again implies (15).

For convenience, we abuse notation a little by denoting the type distribution functions and bid distribution functions in a BNE $\underline{\sigma}$ of $\mathcal{G}(\delta_{\underline{v}_1}, \Phi_2(v_2|a_{2,\underline{\sigma}}))$ by $F_{i,\underline{\sigma}}$ and $H_{i,\underline{\sigma}}$, while in a BNE σ of $\mathcal{G}(\tilde{F}_1, \Phi_2(v_2|a_{2,\sigma}))$ by $F_{i,\sigma}$ and $H_{i,\sigma}$ for any generic $\tilde{F}_1 \neq \delta_{\underline{v}_1}$.

From Lemma 1, in $\underline{\sigma}$,

$$H'_{1,\underline{\sigma}}(\beta) = \frac{1}{F^{-1}_{2,\underline{\sigma}}(H_{2,\underline{\sigma}}(\beta))}, \; H'_{2,\underline{\sigma}}(\beta) = \frac{1}{F^{-1}_{1,\underline{\sigma}}(H_{1,\underline{\sigma}}(\beta))}.$$

Similarly, in σ,

$$H'_{1,\sigma}(\beta) = \frac{1}{F^{-1}_{2,\sigma}(H_{2,\sigma}(\beta))}, \; H'_{2,\sigma}(\beta) = \frac{1}{F^{-1}_{1,\sigma}(H_{1,\sigma}(\beta))}.$$

By Lemma 4, we only need to focus on the case with $c_{2,\sigma} = 0$.

Observe that $F^{-1}_{1,\underline{\sigma}}(H) = \underline{v}_1 \leq F^{-1}_{1,\sigma}(\tilde{H})$ for any H and $\tilde{H}$, which implies $H'_{2,\underline{\sigma}}(\beta) \geq H'_{2,\sigma}(\beta)$ for each β. Because $c_{2,\underline{\sigma}} = c_{2,\sigma} = 0$, $H_{2,\underline{\sigma}}(\beta) \geq H_{2,\sigma}(\beta)$ for each β and $x_\sigma \geq x_{\underline{\sigma}}$ by Lemma 1.

It follows from (15) that $F_{2,\underline{\sigma}}(x) < F_{2,\sigma}(x)$, and thus $F^{-1}_{2,\underline{\sigma}}(H) > F^{-1}_{2,\sigma}(H)$ for each H.[21] Because $F^{-1}_{2,\underline{\sigma}}(H)$ and $F^{-1}_{2,\sigma}(H)$ are both weakly increasing, $F^{-1}_{2,\underline{\sigma}}(H_{2,\underline{\sigma}}(\beta)) > F^{-1}_{2,\sigma}(H_{2,\sigma}(\beta))$. So $H'_{1,\underline{\sigma}}(\beta) < H'_{1,\sigma}(\beta)$. Since $x_\sigma \geq x_{\underline{\sigma}}$, we have $c_{1,\underline{\sigma}} > c_{1,\sigma}$, which contradicts (15) and thus the supposition. Thus $a_{2,\underline{\sigma}} \leq a_{2,\sigma}$.

Now we consider empty rejection set. We explain below that it cannot be the case that for a given b, with belief $v_1 = \underline{v}_1$ the rejection set is empty but with some different belief the rejection set is non-empty. For the convenience of exposition, we use $a_{2,\sigma} = \overline{v}_2$ to mean an empty set

[21] $F_{2,\sigma}(v_2)$ is the conditional distribution $F_2(v_2|v_2 \geq a_{2,\sigma})$. So stochastic dominance is implied.

because if the only rejecting type is $\bar{v}_2$, then rejection is a zero probability event. First, we explain that if, for a given off-path b, the rejection set induced by belief $v_1 = \underline{v}_1$ is empty, i.e., $a_{2,\underline{\sigma}} = \bar{v}_2$, then the rejection set induced by any other belief is also empty, i.e., $a_{2,\sigma} = \bar{v}_2$. To see this, suppose, for a different belief than $v_1 = \underline{v}_1$, the rejection set is non-empty. So σ is not null and $a_{2,\sigma} < \bar{v}_2$. Then the highest type of player 2, i.e., type $\bar{v}_2$, must earn a payoff strictly higher than b in σ by bidding x_σ. Then with belief $v_1 = \underline{v}_1$, type $\bar{v}_2$ can reject the bribe. From above, in any BNE $\underline{\sigma}$ of the continuation auction, $x_{\underline{\sigma}} \leq x_\sigma$. Therefore, type $\bar{v}_2$ at least can bid $x_{\underline{\sigma}}$ and still earn a payoff strictly higher than b, i.e., type $\bar{v}_2$ cannot be indifferent between accepting b and rejecting it. Therefore, the rejection set cannot be empty for belief $v_1 = \underline{v}_1$. So $a_{2,\underline{\sigma}} \leq \bar{v}_2$, which means that if $a_{2,\underline{\sigma}} = \bar{v}_2$ then $a_{2,\sigma} = \bar{v}_2$. Therefore, even when empty rejection set is taken into account, we still have $a_{2,\underline{\sigma}} \leq a_{2,\sigma}$.

A.7 Proof of Theorem 2

Implementability requires that for each off-path bribe b, there exists a belief such that it is not profitable for any type v_1 to deviate to it and in particular not profitable for type $\underline{v}_1$ to deviate to b.

From Lemma 6, for any given b, for any $\tilde{F}_1$, $\pi_1(\underline{v}_1|b,\tilde{F}_1) = F_2(a_{2,\sigma}(b))(\underline{v}_1 - b)$. For any given b, $\pi_1(\underline{v}_1|b,\tilde{F}_1)$ is minimized by the lowest possible $a_{2,\sigma}(b)$ among all possible beliefs $\tilde{F}_1$. By Lemma 8, for any given b, the lowest possible $a_{2,\sigma}(b)$ is achieved by the lowest belief, namely the belief $v_1 = \underline{v}_1$. Thus implementability requires that

$$\underline{v}_1 - \bar{b} \geq \max_b \pi_1(\underline{v}_1|b,\tilde{F}_1). \tag{16}$$

On the other hand, by Lemma 5, the expected payoff of player 1, $\pi_1(v_1|b,\delta_{\underline{v}_1})$, is increasing in v_1 and increases at rates no greater than one everywhere. Thus the condition in (16) is also sufficient for no-profitable-deviation of player 1.

So we can conclude that peace is implementable if and only if there exists a $\bar{b}$ such that (3) and (16) are satisfied, or equivalently, because $U_2(\bar{v}_2|\bar{\sigma}_2) = \bar{v}_2 c_{1,\bar{\sigma}_2}$, the following is satisfied,

$$\bar{v}_2 c_{1,\bar{\sigma}_2} + \max_b \pi_1(\underline{v}_1|b,\delta_{\underline{v}_1}) \leq \underline{v}_1.$$

Next, consider the maximization problem $\max_b \pi_1(\underline{v}_1|b,\delta_{\underline{v}_1})$. According to Lemma 7, if $b \geq \bar{v}_2 - \underline{v}_1$, then b is accepted with probability one. So if $\bar{v}_2 - \underline{v}_1 \leq 0$, then the only possible peaceful bribe is zero and peace is implementable if and only if $\bar{v}_2 c_{1,\bar{\sigma}_2} + F_2(\bar{v}_2)(\underline{v}_1 - 0) \leq \underline{v}_1$, or equivalently

$$c_{1,\bar{\sigma}_2} = 0.$$

If $\bar{v}_2 - \underline{v}_1 > 0$ and $b \geq \bar{v}_2 - \underline{v}_1$, then the highest $\pi_1(\underline{v}_1|b, \delta_{\underline{v}_1})$ among such b is $2\underline{v}_1 - \bar{v}_2$ when $b = \bar{v}_2 - \underline{v}_1$. If $\bar{v}_2 - \underline{v}_1 > 0$ and $b < \bar{v}_2 - \underline{v}_1$, then the rejection set is non-empty and $a_{2,\underline{\sigma}}(b)$ is uniquely given by $1 - \frac{b}{a_{2,\underline{\sigma}}} = \underline{v}_1 \mathcal{I}_2(a_{2,\underline{\sigma}})$. It is clear that $\pi_1(\underline{v}_1|b, \delta_{\underline{v}_1})$ is continuous and the choice set can be restricted to $[0, \bar{v}_2 - \underline{v}_1]$ which is compact. So although in principle there could be multiple stationary points to $\pi_1(\underline{v}_1|b, \delta_{\underline{v}_1})$, a solution and the maximum can be solved by a standard approach.

Finally, we explain the robustness of the equilibrium. Observe that Lemma 5 shows that for any off-path b and any $\tilde{F}_1$, $\pi_1(v_1|b, \tilde{F}_1)$, the expected payoff from a deviation to b, increases continuously at rates no greater than one. This implies that for any off-path b and any $\tilde{F}_1$, if it is profitable for some type v_1 to deviate, then it is also profitable for type $\underline{v}_1$ to deviate, because the equilibrium payoff of player 1 is $v_1 - \bar{v}$ which increases at a rate of one. Thus, the belief $v_1 = \underline{v}_1$ survives the D1 criterion.

Similarly, the expected payoff of any type v_2 from the rejection of the on-path bribe, $U_2(v_2|\sigma_2)$, is increasing for any BNE σ_2 induced any belief $\tilde{F}_2$ whereas the equilibrium payoff of type v_2 is $\bar{b}$. Thus, whenever it is profitable for some type v_2 to reject the equilibrium bribe $\bar{b}$, then it is also profitable for type $\bar{v}_2$ to reject it. Thus, the belief $v_2 = \bar{v}_2$ also survives the D1 criterion.

Therefore, (7) is also the necessary and sufficient condition for the existence of a robust equilibrium in the sense of D1.

A.8 Proof of Corollary 1

Consider the case $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds \leq 1$. Suppose peace is implementable through the zero bribe. Consider the continuation auction $\mathcal{G}(F_1, \bar{v}_2)$ (and a BNE $\bar{\sigma}_2$) following the rejection of the on-path zero bribe. Because $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds \leq 1$, $c_{1,\bar{\sigma}_2} = 0$ and thus $x_{\bar{\sigma}_2} = \bar{v}_2$ by Remark 2. So $U_2(\bar{v}_2|\bar{\sigma}_2) = 0$ and it is not profitable for any type of player 2 to reject the zero bribe given the belief $v_2 = \bar{v}_2$. Since it cannot be profitable for any type v_1 to offer a higher bribe, peace is implementable with the zero bribe.

If $\underline{v}_1 = 0$, then the only possible peaceful equilibrium bribe is zero. If in addition $\bar{v}_2 \int_0^1 (F_1^{-1}(s))^{-1} ds >$ 1, then by Remark 2, $c_{1,\sigma} > 0$ in any BNE σ_2 of the continuation auction $\mathcal{G}(F_1, \delta_{\bar{v}_2})$ following rejection of the on-path zero bribe. Thus $U_2(\bar{v}_2|\bar{\sigma}_2) = \bar{v}_2 c_{1,\sigma} > 0$ and rejection is a profitable deviation (so the condition in (7) fails).

A.9 Proof of Lemma 9

Consider two on-path bribes b_h and $b_l < b_h$. Let the infimum type who offers b_h be denoted by $v_h = \inf\{v_1 : b(v_1) = b_h\}$ and the supremum type who offers b_l be denoted by $v_l = \sup\{v_1 : b(v_1) = b_l\}$. Correspondingly, for $i = h, l$, let a_{2,σ_i} be the lowest rejecting type and σ_i be a BNE of the continuation auction following the rejection of b_i.

We show that type $v_h \geq v_l$ in any equilibrium. To show this, suppose to the contrary $v_h < v_l$ in some equilibrium.

Let $\pi_1(v_i|b_j)$ be the expected payoff of type v_i from offering b_j.

If b_h is accepted with probability one, then any other on-path bribe is rejected with a positive probability. So, any type v_1 higher than v_h can offer b_h and $\pi_1(v_1|b_h)$ increases at a rate of one. On the other hand, because b_l is rejected with a positive probability, the expected payoff of any type $v_1 < v_l$ from offering b_l is

$$\pi_1(v_1|b_l) = F_2(a_{2,\sigma_l}(b_l))(v_1 - b_l) + (1 - F_2(a_{2,\sigma_l}(b_l)))(\max_\beta H_{2,\sigma_l}(\beta)v_1 - \beta)$$

which decreases at rates less than one when v_1 decreases from v_l. So if $\pi_1(v_l|b_l) \geq \pi_1(v_l|b_h)$, then $\pi_1(v_h|b_l) > \pi_1(v_h|b_h)$ and thus b_l is a profitable deviation for type b_h. Hence, in the equilibrium $\pi_1(v_l|b_l) < \pi_1(v_l|b_h)$. But then this means that b_h is a profitable deviation for type v_l. Therefore, b_h cannot be accepted with probability one in the equilibrium.

Obviously, b_l cannot be accepted with probability one in the equilibrium.[22] Hence, below we can focus on the case in which both b_h and b_l are rejected with positive probabilities.

Let $v'_h = \sup\{v_1 : b(v_1) = b_h\}$. Clearly $v'_h \neq v_l$.

Suppose $v'_h < v_l$. Incentive compatibility requires that the expected payoff of type v_l from offering b_h cannot exceed the equilibrium payoff, namely $\pi_1(v_l|b_l) \geq \pi_1(v_l|b_h)$. By Lemma 1, this means that

$$\begin{aligned}
&F_2(a_{2,\sigma_l}(b_l))(v_l - b_l) + (1 - F_2(a_{2,\sigma_l}(b_l)))(v_l - x_{\sigma_l}) \\
\geq &F_2(a_{2,\sigma_h}(b_h))(v_l - b_h) + (1 - F_2(a_{2,\sigma_h}(b_h)))(\max_\beta H_{2,\sigma_h}(\beta)v_l - \beta).
\end{aligned}$$

In σ_h, $x_{\sigma_h} \leq v'_h < v_l$. So x_{σ_h} is an admissible bid for type v_l in the continuation auction and this implies that $\max_\beta H_{2,\sigma_h}(\beta)v_l - \beta \geq v_l - x_{\sigma_h}$. So it follows that

$$\begin{aligned}
&F_2(a_{2,\sigma_l}(b_l))b_l + (1 - F_2(a_{2,\sigma_l}(b_l)))x_{\sigma_l} \\
\leq &F_2(a_{2,\sigma_h}(b_h))b_h + (1 - F_2(a_{2,\sigma_h}(b_h)))x_{\sigma_h}.
\end{aligned} \tag{17}$$

[22] By offering b_l, the payoff of type v_h is $v_h - b_l$. By offering b_h, the expected payoff of type v_h is $F_2(a_{2,\sigma_h}(b_h))(v_h - b_h)$ because $v_h = \inf\{v_1 : b(v_1) = b_h\}$ and $b_h > 0$ implies that $c_{2,\sigma_h} = 0$ in σ_h. Since $b_h > b_l$, $F_2(a_{2,\sigma_h}(b_h))(v_h - b_h) < v_h - b_l$ and it is better for type v_h to offer b_l.

Similarly, incentive compatibility requires that $\pi_1(v'_h|b_h) \geq \pi_1(v'_h|b_l)$. That is,

$$\begin{aligned}&F_2(a_{2,\sigma_h}(b_h))(v'_h - b_h) + (1 - F_2(a_{2,\sigma_h}(b_h))(v'_h - x_{\sigma_h})\\ \geq &F_2(a_{2,\sigma_l}(b_l))(v'_h - b_l) + (1 - F_2(a_{2,\sigma_l}(b_l)))(\max_\beta H_{2,\sigma_l}(\beta)v'_h - \beta).\end{aligned}$$

Because x_{σ_l} is the highest bid in σ_l and $v'_h < v_l$, $\max_\beta H_{2,\sigma_l}(\beta)v'_h - \beta > v'_h - x_{\sigma_l}$.[23] So

$$\begin{aligned}&F_2(a_{2,\sigma_h}(b_h))(v'_h - b_h) + (1 - F_2(a_{2,\sigma_h}(b_h))(v'_h - x_{\sigma_h})\\ > &F_2(a_{2,\sigma_l}(b_l))(v'_h - b_l) + (1 - F_2(a_{2,\sigma_l}(b_l)))(v'_h - x_{\sigma_l}),\end{aligned}$$

or equivalently,

$$\begin{aligned}&F_2(a_{2,\sigma_l}(b_l))b_l + (1 - F_2(a_{2,\sigma_l}(b_l)))x_{\sigma_l}\\ > &F_2(a_{2,\sigma_h}(b_h))b_h + (1 - F_2(a_{2,\sigma_h}(b_h)))x_{\sigma_h}.\end{aligned} \tag{18}$$

So (17) and (18) contradict each other. By similar arguments, a similar contradiction is obtained if $v'_h > v_l$. This completes the proof of $v_h \geq v_l$.

A.10 Proof of Lemma 10

It is straightforward to see that in any equilibrium, at most one on-path bribe can be accepted with probability one. So below we will focus on on-path bribes with a positive probability of rejection.

Suppose there exists an equilibrium in which there is an open interval of type v_1 over which the bribing function is continuously increasing. Then the bribes over the interval are separating. By the similar arguments in footnote 18, we can conclude that upon receiving such a separating bribe $b(v_1)$, player 2's best response is described by an interval $[a_2(v_1), \bar{v}_2]$ for some $a_2(v_1)$, i.e., to accept the bribe if $v_2 < a_2(v_1)$ and reject the bribe otherwise.

Since the interval is open, the on-path bribes over the interval are all strictly positive. Consider the continuation auction following rejection of a bribe $b(v_1)$ from the interval. Type $a_2(v_1)$ is indifferent between the expected payoff from the auction and the positive bribe. Since type $a_2(v_1)$ earns a positive expected payoff in any BNE σ of the auction (equal to $c_{1,\sigma}a_2(v_1)$), it

[23] Observe that in the continuation auction following the rejection of b_l, it is optimal for type v_l to bid x_{σ_l} (i.e., $\max_\beta H_{2,\sigma_l}(\beta)v_l - \beta = v_l - x_{\sigma_l}$) but not optimal for any type $v_1 < v_l$. So the envelope theorem implies $\max_\beta H_{2,\sigma_l}(\beta)v_1 - \beta$ decreases at rates less than one when v_1 decreases from v_l to v'_h. On the other hand, $v_1 - x_{\sigma_l}$ decreases at a rate of one when v_1 decreases from v_l to v'_h.

follows that $c_{1,\sigma} > 0$. So, type v_1 earns zero payoff in the auction and the upper bound of the common bidding interval is v_1.

Consider player 1's another type $\hat{v}_1$ that mimics type v_1 by offering bribe $b(v_1)$. By Lemma 3, if $\hat{v}_1 > v_1$, type $\hat{v}_1$'s optimal choice is to bid v_1 to win for sure and the payoff is thus $\hat{v}_1 - v_1$. Similarly, if $\hat{v}_1 < v_1$, then type $\hat{v}_1$'s optimal choice is to bid zero and the payoff is zero.

Now consider a bribe $b(v_1)$ from type v_1 and another bribe $b(\hat{v}_1)$ from type $\hat{v}_1$ in the neighborhood of v_1.

If type $\hat{v}_1 > v_1$ mimics type v_1 by offering $b(v_1)$, the expected payoff is

$$\pi_1(\hat{v}_1, v_1) = F_2(a_2(v_1))(\hat{v}_1 - b(v_1)) + (1 - F_2(a_2(v_1)))(\hat{v}_1 - v_1).$$

The incentive compatibility condition requires[24]

$$f_2(a_2(v_1))(v_1 - b(v_1))a_2'(v_1) - F_2(a_2(v_1))b'(v_1) - (1 - F_2(a_2(v_1))) = 0. \tag{19}$$

If type $v_1 < \hat{v}_1$ mimics type $\hat{v}_1$ by offering a separating bribe $b(\hat{v}_1)$, the expected payoff is

$$\pi_1(v_1, \hat{v}_1) = F_2(a_2(\hat{v}_1))(v_1 - b(\hat{v}_1)).$$

The incentive compatibility condition requires

$$f_2(a_2(v_1))(v_1 - b(v_1))a_2'(v_1) - F_2(a_2(v_1))b'(v_1) = 0. \tag{20}$$

The conditions (19) and (20) together imply

$$-(1 - F_2(a_2(v_1))) = 0,$$

which can be true if and only if $a_2(v_1) = \bar{v}_2$ for all v_1 and thus $b'(v_1) = 0$, a contradiction to the assumption of the separating segment.

A.11 Proof of Theorem 3

We show the result by showing that there exist no equilibria in which there are two consecutive pooling bribes rejected with positive probabilities. This is because, from Lemma 9 and 10, we know that in any non-peaceful equilibrium, the bribing function can only be a non-decreasing

[24] The condition $\hat{v}_1 \in \arg\max_{\hat{v}_1} \pi_1(\hat{v}_1, v_1)$ yields

$$f_2(a_2(\hat{v}_1))(\hat{v}_1 - b(\hat{v}_1))a_2'(\hat{v}_1) - F_2(a_2(\hat{v}_1))b'(\hat{v}_1) - (1 - F_2(a_2(\hat{v}_1))) = 0.$$

Replacing $\hat{v}_1$ by v_1 we obtain (19).

step function or a constant. So, if no consecutive pooling bribes are rejected with positive probabilities and the equilibrium involves a step function, then there can only be two on-path bribes, with the higher one being accepted with probability one.

Suppose there exists an equilibrium in which any type $v_1 \in [v_l, \hat{v}_1\rangle$ offers a bribe b_l and any type $v_1 \in \langle\hat{v}_1, v_h]$ offers a bribe b_h for some $v_l < \hat{v}_1 < v_h$.[25] By Lemma 9, $b_h > b_l \geq 0$. Suppose further both b_h and b_l are rejected with positive probabilities. Let the rejection set of b_i be $[a_{2,\sigma^i}(b_i), \overline{v_2}]$ for $i = h, l$. And denote a BNE of the continuation auction following rejection of b_i by σ^i and the highest bid by x_{σ^i}.

Clearly type $\hat{v}_1$ is indifferent between b_h and b_l. It follows from Lemma 3 that in σ^h type $\hat{v}_1$ bids zero and the expected payoff is $c_{2,\sigma^h}\hat{v}_1$ whereas in σ^l type $\hat{v}_1$ bids x_{σ^l} and the payoff is $\hat{v}_1 - x_{\sigma^l}$. Because type $a_{2,\sigma^h}(b_h)$ must be indifferent between rejecting $b_h > 0$ and accepting it (and thus earn a positive payoff), it follows that $c_{1,\sigma^h} > 0$ and $c_{2,\sigma^h} = 0$. So the indifference condition for type $\hat{v}_1$ is

$$F_2(a_{2,\sigma^h}(b_h))(\hat{v}_1 - b_h) = F_2(a_{2,\sigma^l}(b_l))(\hat{v}_1 - b_l) + (1 - F_2(a_{2,\sigma^l}(b_l)))(\hat{v}_1 - x_{\sigma^l}).$$

The fact $c_{1,\sigma^h} > 0$ means that a positive measure of types v_1 in the right neighborhood of $\hat{v}_1$ earn zero in σ^h. It follows that the highest bid in σ^h, $x_{\sigma^h} > \hat{v}_1$. So

$$\begin{aligned}
&F_2(a_{2,\sigma^h}(b_h))(\hat{v}_1 - b_h) + (1 - F_2(a_{2,\sigma^h}(b_h)))(\hat{v}_1 - x_{\sigma^h}) \\
<&F_2(a_{2,\sigma^l}(b_l))(\hat{v}_1 - b_l) + (1 - F_2(a_{2,\sigma^l}(b_l)))(\hat{v}_1 - x_{\sigma^l}),
\end{aligned}$$

which is equivalent to

$$F_2(a_{2,\sigma^h}(b_h))b_h + (1 - F_2(a_{2,\sigma^h}(b_h)))x_{\sigma^h} > F_2(a_{2,\sigma^l}(b_l))b_l + (1 - F_2(a_{2,\sigma^l}(b_l)))x_{\sigma^l}. \quad (21)$$

In the equilibrium, the expected payoff of type v_h is

$$\begin{aligned}
\pi_1(v_h) &= F_2(a_{2,\sigma^h}(b_h))(v_h - b_h) + (1 - F_2(a_{2,\sigma^h}(b_h)))(v_h - x_{\sigma^h}) \\
&= v_h - [F_2(a_{2,\sigma^h}(b_h))b_h + (1 - F_2(a_{2,\sigma^h}(b_h)))x_{\sigma^h}].
\end{aligned}$$

Similarly, by deviating to b_l, the expected payoff of type v_h is

$$\pi_1(v_h|b_l) = v_h - [F_2(a_{2,\sigma^l}(b_l))b_l + (1 - F_2(a_{2,\sigma^l}(b_l)))x_{\sigma^l}].$$

So, it follows from (21) that $\pi_1(v_h|b_l) > \pi_1(v_h)$ and thus b_l is a profitable deviation for type v_h, a contradiction.

[25] $[a, b\rangle$ means "$[a, b]$ or $[a, b)$". Similarly, $\langle a, b]$ means "$[a, b]$ or $(a, b]$".

B Proofs for the requesting model

B.1 Proof of Lemma 11

Consider first the case $\bar{v}_1 > 2\underline{v}_2$. In any peaceful equilibrium, $\bar{r} \le \underline{v}_2$ and player 1's payoff is $\bar{r}$. Suppose type $\bar{v}_1$ deviates to an off-path request $r \in (\underline{v}_2, \bar{v}_1/2)$. Upon receiving the off-path request r, no rejecting type v_2 would bid more than r in the continuation auction. So for any belief $\tilde{F}_1$ and any σ, $x_\sigma \le r$. So, $\bar{v}_1 - x_\sigma \ge \bar{v}_1 - r > \bar{v}_1 - \bar{v}_1/2 = \bar{v}_1/2 > \underline{v}_2 \ge \bar{r}$. Since $r > \bar{r}$, for any belief $\tilde{F}_1$ and any σ, type $\bar{v}_1$'s expected payoff $\pi_1(\bar{v}_1|r, \tilde{F}_1) > \bar{r}$ by (8). Therefore, such an off-path $r \in (\underline{v}_2, \bar{v}_1/2)$ is always a profitable deviation for type $\bar{v}_1$. So in this case, there exist no peaceful equilibria.

So, there exists a peaceful equilibrium only if $\bar{v}_1 \le 2\underline{v}_2$. With similar arguments above, it follows that $\bar{r} \ge \bar{v}_1/2$ in any peaceful equilibrium. To see this, suppose to the contrary $\bar{r} < \bar{v}_1/2$. Consider type $\bar{v}_1$'s deviation to $r = \bar{v}_1/2$. Again, in the continuation auction, $x_\sigma \le r = \bar{v}_1/2$ for any beliefs. So $\pi_1(\bar{v}_1|r = \bar{v}_1/2, \tilde{F}_1) \ge \bar{v}_1/2 > \bar{r}$ and thus $r = \bar{v}_1/2$ is a profitable off-path deviation for type $\bar{v}_1$ for any beliefs.

B.2 Proof of Lemma 12

In any robust peaceful equilibrium with D1, for any off-path request, the only reasonable belief is $v_1 = \bar{v}_1$, because the expected payoff of type v_1 from the off-path deviation is increasing. So, we focus on this belief below.

We first show that in this case if $\bar{v}_1 \le \bar{v}_2$ and $\bar{r} < \bar{v}_1$, then type $\bar{v}_1$ can always deviate to some $r \in (\bar{r}, \bar{v}_1)$ which is accepted by all types of player 2 and thus a profitable deviation. To see this, suppose $\bar{r} < \bar{v}_1$. Upon receiving an off-path request $r \in (\bar{r}, \bar{v}_1)$, with belief $v_1 = \bar{v}_1$ the rejection set of player 2 is $[\underline{v}_2, \alpha_{2,\bar{\sigma}}(r)]$ for some $\alpha_{2,\bar{\sigma}}(r) > \underline{v}_2$. In any BNE $\bar{\sigma}$ of $\mathcal{G}(\bar{v}_1, \Psi_2(v_2|\alpha_{2,\bar{\sigma}}(r)))$, $x_{\bar{\sigma}} = \bar{v}_1$ by Lemma 2 because $\bar{v}_1 \le \underline{v}_2$ implies $c_{2,\bar{\sigma}} = 0$.[26] But then this violates the off-path consistency requirement for player 2 because the payoff of type $\alpha_{2,\bar{\sigma}}(r)$ is $\alpha_{2,\bar{\sigma}}(r) - x_{\bar{\sigma}} = \alpha_{2,\bar{\sigma}}(r) - \bar{v}_1$ and thus the fact that $r < \bar{v}_1$ implies that it is better for type $\alpha_{2,\bar{\sigma}}(r)$ to accept r. Therefore, the consistent reply of player 2 is to accept r for all types v_2. But full acceptance implies then that this off-path $r \in (\bar{r}, \bar{v}_1)$ is a profitable deviation for type $\bar{v}_1$.

So, if $\bar{v}_1 \le \bar{v}_2$, the only possible robust peaceful equilibrium request is $\bar{r} = \bar{v}_1$. Suppose such an equilibrium exists and thus in the equilibrium player 2's payoff is $v_2 - \bar{v}_1$. Consider player

[26] If $c_{2,\bar{\sigma}} > 0$, then it means that there is a positive measure of type v_2 earning zero payoff by bidding zero in $\bar{\sigma}$; but any type $v_2 > \bar{v}_1$ can secure a positive payoff by bidding $\bar{v}_1$.

2's deviation–rejection of $\bar{r}$. For any given $\tilde{F}_2$, in any BNE σ_2 of $\mathcal{G}(F_1, \tilde{F}_2)$, because $\bar{v}_1 > \underline{v}_1$, we must have that $U_1(\bar{v}_1|\sigma_2) = \bar{v}_1 - x_{\sigma_2} > 0$, i.e., $x_{\sigma_2} < \bar{v}_1$ (if $U_1(\bar{v}_1|\sigma_2) = 0$, then $U_1(\underline{v}_1|\sigma_2) < 0$ because $U_1(v_1|\sigma_2)$ is strictly decreasing in the neighborhood of $\bar{v}_1$, thus a contradiction). It follows then that the payoff of type $v_2^m :=$ sup supp $\tilde{F}_2$ is $U_2(v_2^m|\sigma_2) = v_2^m - x_{\sigma_2} > v_2^m - \bar{v}_1$. So, rejection of $\bar{r} = \bar{v}_1$ is a profitable deviation and thus a contradiction. This completes the proof.

B.3 Proof of Theorem 4

In any robust peaceful equilibrium with D1, for any off-path request, the only reasonable belief is $v_1 = \bar{v}_1$, because the expected payoff of type v_1 from the off-path deviation is increasing. So, we focus on this belief below.

We first show that in any robust peaceful equilibria $\bar{r} = \underline{v}_2$. Suppose to the contrary $\bar{r} < \underline{v}_2$. Upon receiving an off-path $r \in (\bar{r}, \underline{v}_2)$, with belief $v_1 = \bar{v}_1$ the rejection set of player 2 is $[\underline{v}_2, \alpha_{2,\bar{\sigma}}(r)]$ for some $\alpha_{2,\bar{\sigma}}(r) > \underline{v}_2$. Consider a BNE $\bar{\sigma}$ of $\mathcal{G}(\delta_{\bar{v}_1}, \Psi_2(v_2|\alpha_{2,\bar{\sigma}}(r)))$. If $c_{1,\bar{\sigma}} > 0$, then in $\bar{\sigma}$ type $\bar{v}_1$'s payoff is zero and thus $x_{\bar{\sigma}} = \bar{v}_1$; but then type $\alpha_{2,\bar{\sigma}}(r)$ would find it better to accept r because $r < \underline{v}_2 < \bar{v}_1$. If $c_{1,\bar{\sigma}} = 0$, then in $\bar{\sigma}$ type $\underline{v}_2$'s payoff is zero and thus would find it better to accept r. Therefore, the consistent reply of player 2 receiving such an r is to accept it for all types v_2. But then full acceptance implies that such an r is a profitable deviation for player 1. Therefore, we can conclude that in any robust peaceful equilibria $\bar{r} = \underline{v}_2$.

The arguments above also show that with $\bar{r} = \underline{v}_2$, it cannot be profitable for player 1 to deviate to any off-path $r < \underline{v}_2$ because the consistent reply of player 2 is full acceptance. So below we can focus on $r > \bar{r} = \underline{v}_2$ and clearly for these off-path requests the rejection set is non-empty, i.e., $\alpha_{2,\bar{\sigma}}(r) > \underline{v}_2$.

Recall that $x_{\bar{\sigma}*}$ is the highest bid in any BNE $\bar{\sigma}^*$ of $\mathcal{G}(\delta_{\bar{v}_1}, F_2)$. So when $\bar{v}_1 > \underline{v}_2$, it must be that $x_{\bar{\sigma}^*} \geq \underline{v}_2$. This is because, if $x_{\bar{\sigma}^*} < \underline{v}_2$, then both type $\bar{v}_1$ and type $\underline{v}_2$ win with positive expected payoffs (and thus probabilities) and thus $c_{1,\bar{\sigma}^*}, c_{2,\bar{\sigma}^*} > 0$, which is impossible.

Consider first an $r > x_{\bar{\sigma}^*}$. We show that for such an r the consistent reply of player 2 is full rejection and thus it is not profitable for type $\bar{v}_1$ to deviate to such an r. To see this, suppose that the rejection set is $[\underline{v}_2, \alpha_{2,\bar{\sigma}}(r)]$ and $\alpha_{2,\bar{\sigma}}(r) < \bar{v}_2$. Then since $\alpha_{2,\bar{\sigma}}(r)$ is in the interior of the support of F_2, the indifference condition implies that in any BNE $\bar{\sigma}$ of the continuation auction $\mathcal{G}(\delta_{\bar{v}_1}, \Psi_2(v_2|\alpha_{2,\bar{\sigma}}(r)))$, $x_{\bar{\sigma}} = r > x_{\bar{\sigma}^*}$. From Lemma 2, $x_{\bar{\sigma}} = \bar{v}_1(1 - c_{2,\bar{\sigma}})$. Because $x_{\bar{\sigma}^*} = \bar{v}_1(1 - c_{2,\bar{\sigma}^*})$, we have $c_{2,\bar{\sigma}^*} > c_{2,\bar{\sigma}} \geq 0$. Because $c_{2,\bar{\sigma}^*} > 0$ and by definition of $c_{2,\bar{\sigma}^*}$ we have $\bar{v}_1 \int_{c_{2,\bar{\sigma}^*}}^1 \left(F_2^{-1}(s)\right)^{-1} ds = 1$, it follows that

$$\bar{v}_1 \int_{c_{2,\bar{\sigma}}}^1 \left(F_2^{-1}(s)\right)^{-1} ds > 1$$

which in turn implies

$$c_{1,\bar{\sigma}} = 1 - \bar{v}_1 \int_{c_{2,\bar{\sigma}}}^{1} \left(F_2^{-1}(s)\right)^{-1} ds < 0$$

which is impossible. Therefore, for any $r > x_{\bar{\sigma}^*}$, the consistent reply of player 2 is full rejection, i.e., $\alpha_{2,\bar{\sigma}}(r) = \bar{v}_2$. From above, in any BNE $\bar{\sigma}^*$ of $\mathcal{G}(\delta_{\bar{v}_1}, F_2)$, $x_{\bar{\sigma}^*} \geq \underline{v}_2$. Because $\bar{v}_1 \leq 2\underline{v}_2$, type $\bar{v}_1$'s payoff is $\bar{v}_1 - x_{\bar{\sigma}^*} \leq \underline{v}_2$. So with full rejection, it is not profitable for type $\bar{v}_1$ to deviate to such an r.

Next consider an $r \in (\underline{v}_2, x_{\bar{\sigma}^*})$ and the rejection set $[\underline{v}_2, \alpha_{2,\bar{\sigma}}(r)]$. If full rejection is a consistent reply, i.e., $\alpha_{2,\bar{\sigma}}(r) = \bar{v}_2$, then the continuation auction is $\mathcal{G}(\delta_{\bar{v}_1}, F_2)$ and the highest bid in any BNE $\bar{\sigma}^*$ is $x_{\bar{\sigma}^*}$. Because $r < x_{\bar{\sigma}^*}$, in $\bar{\sigma}^*$ the payoff of type $\bar{v}_2$, i.e., $\bar{v}_2 - x_{\bar{\sigma}^*}$, is lower than the payoff from acceptance, i.e., full rejection violates the off-path consistency requirement. Therefore, for any $r \in (\underline{v}_2, x_{\bar{\sigma}^*})$, $\alpha_{2,\bar{\sigma}}(r) < \bar{v}_2$ in any consistent reply. So the indifference condition for type $\alpha_{2,\bar{\sigma}}(r)$ implies: there is a unique consistent reply with $\alpha_{2,\bar{\sigma}}(r)$ given by (9);[27] the highest bid in any BNE $\bar{\sigma}$ of the continuation auction $x_{\bar{\sigma}} = r$ and thus the expected payoff of type $\bar{v}_1$ is $F_2(\alpha_{2,\bar{\sigma}}(r))(\bar{v}_1 - r) + (1 - F_2(\alpha_{2,\bar{\sigma}}(r)))r$.

We can then formulate the maximization problem in (10) with the compact choice set including $\underline{v}_2$ and $x_{\bar{\sigma}^*}$. So in any robust peaceful equilibrium, it is not profitable for any type v_1 to deviate to any off-path request if and only if $\underline{v}_2 \geq F_2(\alpha_{2,\bar{\sigma}}(r^*))(\bar{v}_1 - r^*) + (1 - F_2(\alpha_{2,\bar{\sigma}}(r^*)))r^*$. Then by continuity of the objective function in (10) the inequality condition is equivalent to the equality condition.

Now we consider player 2's rejection of the on-path request. In any robust peaceful equilibrium with $\bar{r} = \underline{v}_2$, if the equilibrium request $\bar{r}$ is rejected, the only reasonable belief about v_2 is that $v_2 = \underline{v}_2$ because for any belief the expected payoff of player 2 increases at rates no greater than one. Thus it is not profitable for any type v_2 to reject $\bar{r}$ if and only if $U_2(\underline{v}_2|\underline{\sigma}_2) = 0$, namely $\underline{v}_2 c_{1,\underline{\sigma}_2} = 0$.

Aggregating all the results above, the proof is completed.

B.4 Proof of Lemma 13

We first show that $\alpha_{2,\underline{\sigma}} \geq \alpha_{2,\sigma}$. To show this, suppose to the contrary, $\alpha_{2,\underline{\sigma}} < \alpha_{2,\sigma}$.

Given an off-path bribe r, consider the BNE $\underline{\sigma}$ of $\mathcal{G}(\delta_{\underline{v}_1}, \Psi_2(v_2|\alpha_{2,\underline{\sigma}}))$ induced by belief $v_1 = \underline{v}_1$ and some BNE σ of $\mathcal{G}(\tilde{F}_1, \Psi_2(v_2|\alpha_{2,\sigma}))$ induced by some belief $\tilde{F}_1$. If $\alpha_{2,\sigma} < \bar{v}_2$, then

[27] Since $\bar{v}_1 \geq x_{\bar{\sigma}^*} > r = x_{\bar{\sigma}}$ and thus type $\bar{v}_1$ earns a positive expected payoff, $c_{2,\bar{\sigma}} > 0$ and $c_{1,\bar{\sigma}} = 0$. Applying $c_{1,\bar{\sigma}} = 0$ to (2) for $\mathcal{G}(\delta_{\bar{v}_1}, \Psi_2(v_2|\alpha_{2,\bar{\sigma}}(r)))$, it is clear that $\alpha_{2,\underline{\sigma}}(r)$ is given by (9).

$x_\sigma = x_{\underline{\sigma}} = r$. If $\alpha_{2,\sigma} < \bar{v}_2$, then $x_\sigma \le x_{\underline{\sigma}} = r$. Hence, $\alpha_{2,\underline{\sigma}} < \alpha_{2,\sigma}$ implies

$$x_\sigma \le x_{\underline{\sigma}}.$$

For convenience, we abuse notation a little by denoting the type distribution functions $\tilde{F}_i$ and bid distribution functions $\tilde{H}_i$ in the BNE $\underline{\sigma}$ by $F_{i,\underline{\sigma}}$ and $H_{i,\underline{\sigma}}$, while in the BNE σ by $F_{i,\sigma}$ and $H_{i,\sigma}$ for any generic $\tilde{F}_1 \neq \delta_{\underline{v}_1}$.

From Lemma 1, in $\underline{\sigma}$,

$$H'_{1,\underline{\sigma}}(\beta) = \frac{1}{F^{-1}_{2,\underline{\sigma}}(H_{2,\underline{\sigma}}(\beta))},\ H'_{2,\underline{\sigma}}(\beta) = \frac{1}{F^{-1}_{1,\underline{\sigma}}(H_{1,\underline{\sigma}}(\beta))}.$$

Similarly, in σ,

$$H'_{1,\sigma}(\beta) = \frac{1}{F^{-1}_{2,\sigma}(H_{2,\sigma}(\beta))},\ H'_{2,\sigma}(\beta) = \frac{1}{F^{-1}_{1,\sigma}(H_{1,\sigma}(\beta))}.$$

Observe that $F^{-1}_{1,\underline{\sigma}}(H) = \underline{v}_1 \le F^{-1}_{1,\sigma}\left(\tilde{H}\right)$ for any H and $\tilde{H}$, which implies $H'_{2,\underline{\sigma}}(\beta) \ge H'_{2,\sigma}(\beta)$ for each β. Because $x_\sigma \le x_{\underline{\sigma}}$,

$$H_{2,\underline{\sigma}}(\beta) = 1 - \int_\beta^{x_{\underline{\sigma}}} H'_{2,\underline{\sigma}}(x)dx \le 1 - \int_\beta^{x_\sigma} H'_{2,\sigma}(x)dx = H_{2,\sigma}(\beta)$$

for any β. In particular, $c_{2,\underline{\sigma}} = H_{2,\underline{\sigma}}(0) \le H_{2,\sigma}(0) = c_{2,\sigma}$. Because $c_{1,\underline{\sigma}}c_{2,\underline{\sigma}} = 0 = c_{1,\sigma}c_{2,\sigma}$, we have $c_{1,\underline{\sigma}} \ge c_{1,\sigma}$.

It also follows from $\alpha_{2,\underline{\sigma}} < \alpha_{2,\sigma}$ that $F_{2,\underline{\sigma}}(v_2) > F_{2,\sigma}(v_2)$ for $v_2 \neq \{\underline{v}_2, \alpha_{2,\sigma}\}$, and thus $F^{-1}_{2,\underline{\sigma}}(H) < F^{-1}_{2,\sigma}(H)$ for each $H \neq 0$.[28] Because $F^{-1}_{2,\underline{\sigma}}(H)$ and $F^{-1}_{2,\sigma}(H)$ are both weakly increasing and $H_{2,\underline{\sigma}}(\beta) \le H_{2,\sigma}(\beta)$ for any β, $F^{-1}_{2,\underline{\sigma}}(H_{2,\underline{\sigma}}(\beta)) < F^{-1}_{2,\sigma}(H_{2,\sigma}(\beta))$. So $H'_{1,\underline{\sigma}}(\beta) > H'_{1,\sigma}(\beta)$ for any $\beta > 0$. Since $x_\sigma \le x_{\underline{\sigma}}$, we have

$$c_{1,\underline{\sigma}} = 1 - \int_0^{x_{\underline{\sigma}}} H'_{1,\underline{\sigma}}(\beta)d\beta < 1 - \int_0^{x_\sigma} H'_{1,\sigma}(\beta)d\beta = c_{1,\sigma},$$

a contradiction to $c_{1,\underline{\sigma}} \ge c_{1,\sigma}$ from above. Hence the supposition is false and we have $\alpha_{2,\underline{\sigma}} \ge \alpha_{2,\sigma}$, which also implies $x_{\underline{\sigma}} \le x_\sigma$.

The proof for $\alpha_{2,\sigma} \ge \alpha_{2,\bar{\sigma}}$ and $x_\sigma \le x_{\bar{\sigma}}$ can be done in the same spirit and thus is omitted.

B.5 Proof of Theorem 5

We first consider player 2's rejection of the on-path request $\bar{r}$. Clearly among all possible beliefs and the BNE induced by the continuation auction $\mathcal{G}(F_1, \tilde{F}_2)$, the highest payoff is achieved when

[28] $F_{2,\sigma}(v_2)$ is the conditional distribution $F_2(v_2|v_2 \le \alpha_{2,\sigma})$. So stochastic dominance is implied.

player 1's belief is $\tilde{F}_2 = \delta_{\bar{v}_2}$. Hence it is not profitable for type $\underline{v}_2$ to reject $\bar{r}$ if and only if $\underline{v}_2 c_{1,\bar{\sigma}_2} = 0$.

Below we turn to deviation of player 1 to off-path r.

Suppose first $\underline{v}_1 \geq \underline{v}_2$.

Consider first $r < \underline{v}_2$. For any belief $\tilde{F}_1$, the consistent reply of player 2 receiving such an r is to accept it for all types v_2. To see this, recall from Remark 6 that If full acceptance is the consistent reply to r for belief $v_1 = \underline{v}_1$, then it is also the consistent reply to r for any beliefs. So suppose that for belief $v_1 = \underline{v}_1$ the request r is rejected with a positive probability and consider a BNE $\underline{\sigma}$ of the induced continuation auction $\mathcal{G}(\delta_{\underline{v}_1}, \Psi_2(v_2|\alpha_{2,\underline{\sigma}}(r)))$. In $\underline{\sigma}$, if $c_{1,\underline{\sigma}} > 0$, then in $\underline{\sigma}$ type $\underline{v}_1$'s payoff is zero and thus $x_{\underline{\sigma}} = \underline{v}_1$; but then type $\alpha_{2,\underline{\sigma}}(r)$ would find it better to accept r because $r < \underline{v}_2 < \underline{v}_1$. If $c_{1,\underline{\sigma}} = 0$, then in $\underline{\sigma}$ type $\underline{v}_2$'s payoff is zero and thus would find it better to accept r. So for the belief $v_1 = \underline{v}_1$ and thus for any beliefs the consistent reply to $r < \underline{v}_2$ is full acceptance.

It also follows that it cannot be profitable for player 1 to deviate to any off-path $r < \underline{v}_2$ because the consistent reply of player 2 is full acceptance. So below we can focus on $r > \underline{v}_2$ and clearly for these off-path requests the rejection set is non-empty, i.e., $\alpha_{2,\underline{\sigma}}(r) > \underline{v}_2$.

Consider next $r > \underline{v}_2$. Such an r must lead to a positive probability of rejection.

By Lemma 13 we have $x_{\underline{\sigma}} \leq x_{\sigma}$ for any BNE σ of the continuation auction induced by any belief $\tilde{F}_1$. If the consistent reply is full rejection for some belief, then in any σ of the continuation auction the payoff of type $\bar{v}_1$ is $\bar{v}_1 - x_{\sigma}$. So among all possible beliefs such that full rejection is the consistent reply, the highest payoff of type $\bar{v}_1$ is achieved when the belief is $v_1 = \underline{v}_1$. Let $x_{\underline{\sigma}*}$ be the highest bid in any BNE $\underline{\sigma}^*$ of $\mathcal{G}(\delta_{\underline{v}_1}, F_2)$. So when $\underline{v}_1 > \underline{v}_2$, it must be that

$$x_{\underline{\sigma}^*} \geq \underline{v}_2.$$

This is because, if $x_{\underline{\sigma}^*} < \underline{v}_2$, then both type $\underline{v}_1$ and type $\underline{v}_2$ win with positive probabilities and thus $c_{1,\underline{\sigma}^*}, c_{2,\underline{\sigma}^*} > 0$, which is impossible. Because $\bar{v}_1 \leq 2\underline{v}_2$, type $\bar{v}_1$'s payoff is $\bar{v}_1 - x_{\underline{\sigma}^*} \leq \underline{v}_2$. So for any beliefs inducing full rejection, it is not profitable for type $\bar{v}_1$ to deviate to $r > \underline{v}_2$.

For any belief that induces partial rejection, the expected payoff of type $\bar{v}_1$ is $F_2(\alpha_{2,\sigma}(r))(\bar{v}_1 - r) + (1 - F_2(\alpha_{2,\sigma}(r)))r$. Because $\bar{v}_1 \leq 2\underline{v}_2$ and $r > \underline{v}_2$ implies $\bar{v}_1 - r < r$, the highest expected payoff is achieved by the highest possible $\alpha_{2,\sigma}(r)$ among all σ, namely $\alpha_{2,\bar{\sigma}}(r)$ by Lemma 13. Robustness of the peaceful equilibrium ensures that it is not profitable for type $\bar{v}_1$ to deviate to such r for the belief $v_1 = \bar{v}_1$. So now we are done with $r \geq \underline{v}_2$ and thus the case $\underline{v}_1 > \underline{v}_2$.

Suppose now $\underline{v}_1 \leq \underline{v}_2$.

We first consider the consistent reply to any r for belief $v_1 = \underline{v}_1$. If $r < \underline{v}_1$, then r is accepted by all types of player 2 for the belief $v_1 = \underline{v}_1$. This is because, if r is rejected by some types of player 2, then in the continuation auction the highest bid must be $\underline{v}_1$ and thus the highest rejection type would rather have accepted r. If $r > \underline{v}_1$, then obviously with belief $v_1 = \underline{v}_1$ full rejection is the consistent reply since $v_1 \le \underline{v}_2$ and thus by bidding v_1 in the continuation auction any type v_2 can secure a payoff $v_2 - \underline{v}_1 > v_2 - r$. And in any BNE $\underline{\sigma}$ of the continuation auction $\mathcal{G}(\delta_{\underline{v}_1}, F_2)$ following full rejection, the highest bid is $x_{\underline{\sigma}} = \underline{v}_1$. This is because, if $x_{\underline{\sigma}} < \underline{v}_1 \le \underline{v}_2$, then both type $\underline{v}_1$ and $\underline{v}_2$ earn positive payoffs and thus $c_{1,\underline{\sigma}}, c_{2,\underline{\sigma}} > 0$ which is impossible. It is now also clear that for $r = \underline{v}_1$, player 2 is indifferent between acceptance and rejection for the belief $v_1 = \underline{v}_1$.

We next explain that $\bar{v}_1 - \underline{v}_1 \le \underline{v}_2$ is a necessary condition for peace security. Obviously player 1 can always deviate to a high enough r (e.g., $r > \bar{v}_1$) which leads to full rejection for any beliefs. In particular, with belief $v_1 = \underline{v}_1$, the highest bid in the continuation auction is $\underline{v}_1$ and the payoff of type $\bar{v}_1$ is $\bar{v}_1 - \underline{v}_1$. So peace security requires $\bar{v}_1 - \underline{v}_1 \le \underline{v}_2$.

We now show that $\bar{v}_1 - \underline{v}_1 \le \underline{v}_2$ is also sufficient to ensure that it is not profitable for type $\bar{v}_1$ to deviate to any off-path r for any beliefs.

From Remark 6 we have that if full rejection is the consistent reply to r for some belief, then full rejection is also the consistent reply to r for the belief $v_1 = \underline{v}_1$. So for any given r, among all beliefs that lead to full rejection, the highest payoff of type $\bar{v}_1$ is achieved when the belief is $v_1 = \underline{v}_1$ by Lemma 13. Thus the condition $\bar{v}_1 - \underline{v}_1 \le \underline{v}_2$ also ensures no profitable deviation to r for those beliefs. Hence, for any given r, below we only need to focus on beliefs that lead to partial rejection or full acceptance.

With partial rejection, the expected payoff of type $\bar{v}_1$ is

$$\pi_1(\bar{v}_1 | \alpha_{2,\sigma}(r) \in (\underline{v}_2, \bar{v}_2)) = F_2(\alpha_{2,\sigma}(r))(\bar{v}_1 - r) + (1 - F_2(\alpha_{2,\sigma}(r)))r \tag{22}$$

which is an average of $\bar{v}_1 - r$ and r.

Recall that $\bar{v}_1 \le 2\underline{v}_2$. So $\underline{v}_2 \ge \bar{v}_1/2$.

Consider $r > \underline{v}_2$. Obviously, such an r always leads to a positive probability of rejection for any beliefs. For any belief that induces partial rejection, the expected payoff of type $\bar{v}_1$ is given by (22). Because $\bar{v}_1 - r < \underline{v}_2 < r$, the maximum of $\pi_1(\bar{v}_1 | \alpha_{2,\sigma}(r) \in (\underline{v}_2, \bar{v}_2))$ is achieved by the lowest possible $\alpha_{2,\sigma}(r)$, namely $\alpha_{2,\bar{\sigma}}(r)$ when the belief is $v_1 = \bar{v}_1$. Robustness of the peaceful equilibrium ensures that it is not profitable for type $\bar{v}_1$ to deviate to such r for the belief $v_1 = \bar{v}_1$. So, we are done with $r > \underline{v}_2$.

For any $r \le \underline{v}_2$, full acceptance means no profitable deviation for type $\bar{v}_1$, and thus below we only need to focus on beliefs that lead to partial rejection.

Furthermore, from Remark 6, if for some r the consistent reply is full acceptance for the belief $v_1 = \underline{v}_1$, then full acceptance is the consistent reply for any beliefs. Also from above, for the belief $v_1 = \underline{v}_1$ any request no greater than $\underline{v}_1$ leads to full acceptance. So the request leading to partial rejection must be greater than $\underline{v}_1$. So, below we can assume $r > \underline{v}_1$, which may lead to partial rejection for some beliefs.

Consider $r \in [\underline{v}_1, \underline{v}_2]$. For any belief that induces partial rejection and the BNE σ, the expected payoff of type $\bar{v}_1$ is given by (22). If $r \in [\bar{v}_1/2, \underline{v}_2]$, then because $\bar{v}_1 - r < r \leq \underline{v}_2$, the expected payoff of type $\bar{v}_1$ is not greater than $\underline{v}_2$. So it is not profitable for type $\bar{v}_1$ to deviate to such r for any such beliefs. If $r < \bar{v}_1/2$, then because $r > \underline{v}_1$, we have $\bar{v}_1 - r < \bar{v}_1 - \underline{v}_1$ and thus

$$F_2(\alpha_{2,\sigma}(r))(\bar{v}_1 - r) + (1 - F_2(\alpha_{2,\sigma}(r)))r < F_2(\alpha_{2,\sigma}(r))(\bar{v}_1 - \underline{v}_1) + (1 - F_2(\alpha_{2,\sigma}(r)))r.$$

Since $r < \bar{v}_1/2 \leq \underline{v}_2$, the condition $\bar{v}_1 - \underline{v}_1 \leq \underline{v}_2$ ensures $\pi_1(\bar{v}_1|\alpha_{2,\sigma}(r) \in (\underline{v}_2, \bar{v}_2)) < \underline{v}_2$ and thus it is not profitable for type $\bar{v}_1$ to deviate to such r for any such beliefs.

Aggregating the above, the sufficiency of condition $\bar{v}_1 - \underline{v}_1 \leq \underline{v}_2$ for the case of $\underline{v}_1 \leq \underline{v}_2$ is proved. Therefore, the proof for the theorem is completed.

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